

University Physics I Laboratory

Phys 2125 – Blinn College

Lab Manual

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Lab 0: Graphical Analysis

Equipment: Excel.

Theory (Always READ this section before you do the lab!)

This section will describe the basic procedures for creating scatter plots in Excel and finding a fit to those functions. We will break this down into four basic steps:

- Step 1: Entering the data in the proper order
- Step 2: Creating a scatter plot
- Step 3: Creating Axis Labels and a Graph Title
- Step 4: Creating a fit line or curve.

Step 1: Entering the Data

A common source of error when graphing occurs from two similarly related ideas. What is supposed to be on the vertical and horizontal axes, and which columns should be which in Excel. The standard in the US is to plot vertical versus horizontal. When entering data into excel, the LEFT column represents the horizontal data while the right column represents the vertical data. While this may at first seem backwards, consider this: we say y versus x and yet we plot coordinates (x, y) . Excel follows this notation.

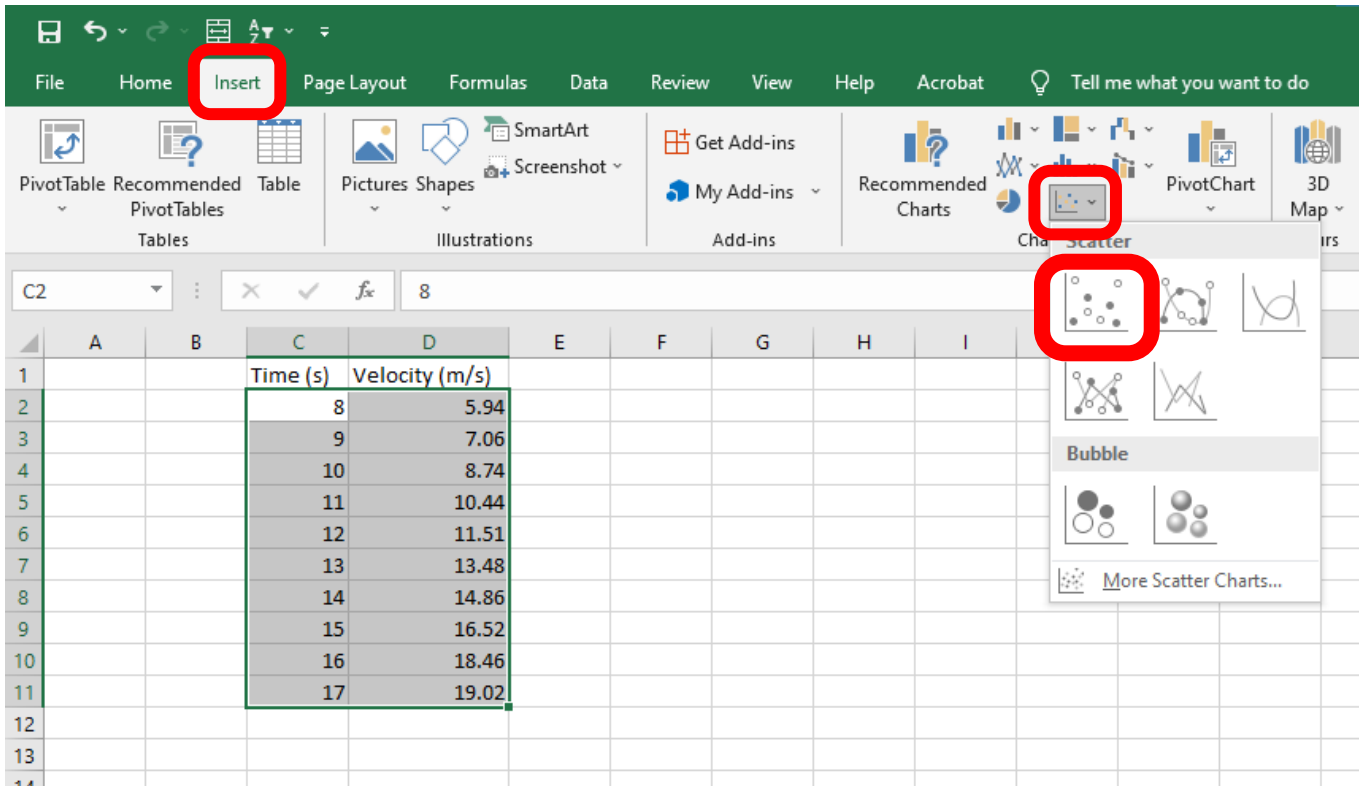
So in a graph of velocity versus time, for instance, the velocity should be represented on the vertical axis and time should be on the horizontal axis. To enter the data into Excel for a plot of velocity versus time, we need the left column to represent the time (horizontal data) and the right column to represent the velocity (vertical data). An example of this is shown here.

	A	B	C	D	E
1			Time (s)	Velocity (m/s)	
2			8	5.94	
3			9	7.06	
4			10	8.74	
5			11	10.44	
6			12	11.51	
7			13	13.48	
8			14	14.86	
9			15	16.52	
10			16	18.46	
11			17	19.02	
12					

Step 2: Creating a scatter Plot

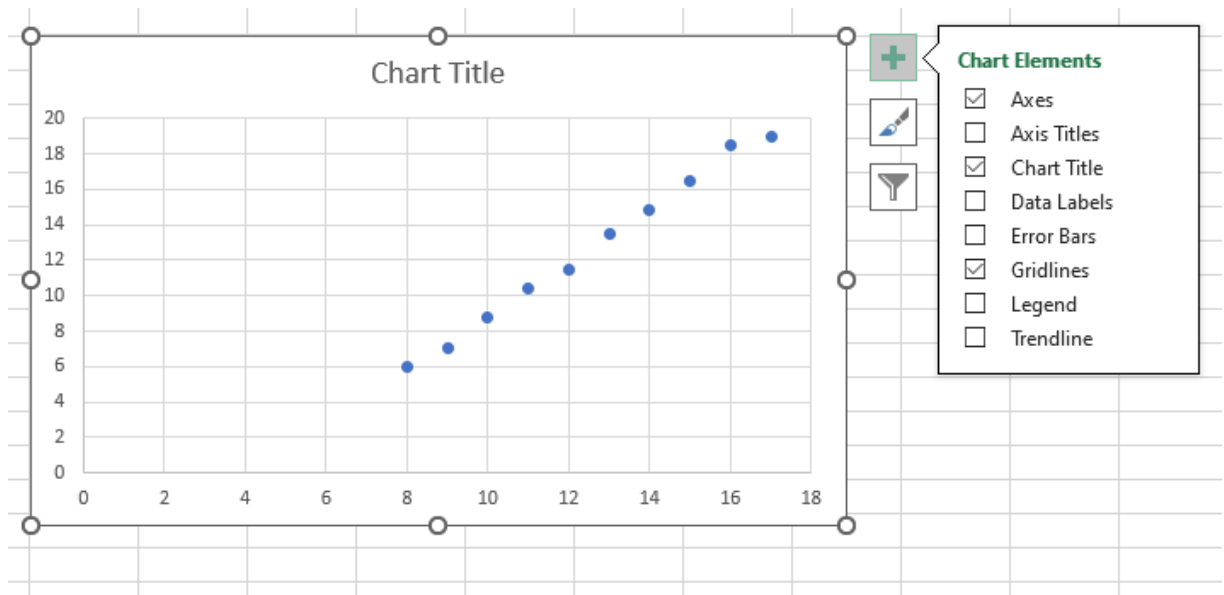
The next step is to have Excel create a plot of the data. The types of plots we will always create in these labs are called scatter plots. To create this you want to highlight all of the data in both columns that you wish to plot. You then want to choose the Insert tab, near the top-left of Excel (shown in the red box). In the picture below,

you can see that we have highlighted the data we are selecting (green box) and we are selecting the buttons to create a scatter plot. (Please do NOT use the connected data points. This will obscure your fit curve.)



Step 3: Creating Axis Labels and a Graph Title

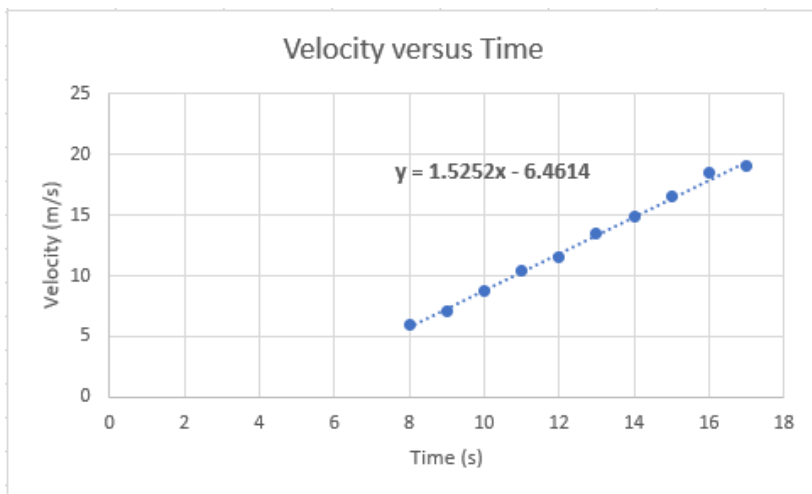
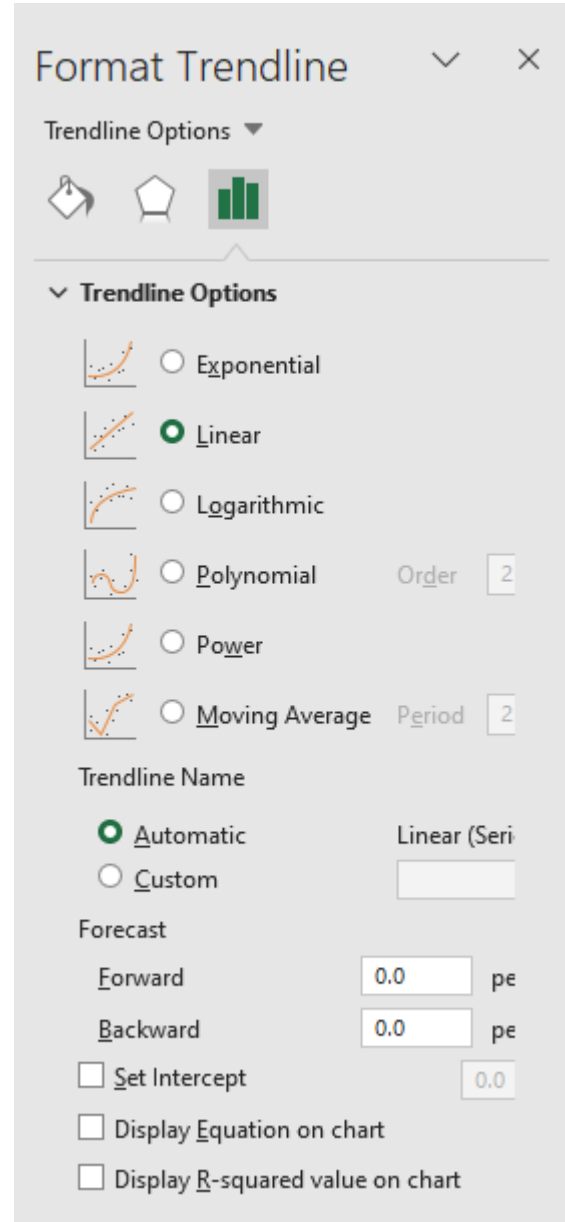
Once step 2 is complete, Excel will create a scatter plot for you which is shown in the figure below. Excel defaults to creating a title for the graph called "Chart Title." You should double click this and change it to something appropriate. To create axis labels, you need to click somewhere on the graph then click the plus sign in the upper right corner when it shows up. This brings up the list of options you can see in the figure. You want to select the "Axis Titles" checkbox which will cause text boxes to show up, which you can edit accordingly.



Step 4: Creating Axis Labels and a Graph Title

It is important to note that this step CANNOT be completed when using Excel online. All Blinn students have access to download and install Excel free of charge. Excel is not installed on all physics lab computers.

You may notice there is a Trend line option in the previous figure. We do not want to create a trend line that way because Excel defaults that to a linear function. Nature and graphs that represent nature's behavior are not always represented as lines, but sometimes as curves. To be able to select the type of function you want to, right-click on one of the data points in the plot. This will bring up a few options, one of which is "Add Trendline." When you click on that, it will bring up a panel on the right side of Excel which looks like the picture to the right. You'll notice it defaults to linear, which is fine if that is representative of the data, as in the example we've been discussing. However, if the data is quadratic, for instance, select Polynomial Order 2 (a cubic function would be Order 3). While not necessarily common in first semester physics, second semester physics sees a number of dependencies which are exponential or logarithmic. These would be what you would want to select if that was representative of your data. One vital step is to make sure that you select the "Display Equation on chart" box. This will provide the formula for the resulting fit on the graph. With all the steps complete the graph above graph would look like this.



Finally, remember, whenever you are asked to create a graph in Excel, you must always:

Include the graph with your lab report!

Part A. Velocity as a function of time with constant acceleration.

Given the following data, plot a graph of v versus t and apply a linear fit.

t (s)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
v (m/s)	3.5	5.0	5.5	6.8	7.5	8.0	10.0	10.2	11.0

In Chapter 2 you will be introduced to an equation that relates v and t when there is constant acceleration. That equation is given below.

$$v = v_0 + at$$

Questions

Based on the equation and the fit on your graph answer the following questions:

A-1. What is the acceleration of the object? Be sure to include the appropriate units.

A-2. What is the velocity at the time defined to be zero? Be sure to include the appropriate units.

Part B. Density

Suppose a beaker is placed on a balance (scale) and different volumes of a liquid are poured into it. Furthermore, suppose the mass of the beaker plus the liquid is recorded each time, and the following data are collected.

<i>Volume</i> (mL)	23	34	45	49	68
<i>Mass</i> (g)	192.8	204.0	219.6	236.5	246.3

Given this data, create a graph of mass versus volume and include a best-fit line.

Questions

B-1. What will the total mass be if there was 50 mL of liquid in the beaker? Show any calculations. Include the appropriate units.

B-2. What is the mass of the beaker? Include appropriate units.

B-3. What is the average density of the liquid? Include appropriate units.

Part C. Fitting Non-linear Relationships.

In the middle part of the semester we will deal with rotational motion. One of the common equations we will deal with in that unit is the following:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

where θ is the angle in radians (rad) at a particular time, θ_0 is the angle at the time defined to be zero, ω_0 is the angular speed at the time defined to be zero, α is the angular acceleration and t is the time. In this case, time has units of seconds (s). You do not need to know anything else about this equation at this time to perform this section. Using the set of data below, create a plot of angle in radians versus time (in seconds) and fit the it with a quadratic.

Angle (rad)	1.480	1.921	2.291	2.495	1.983	1.464	1.017	0.03642	-1.063	-2.517
Time (ms)	5.2	50.8	96.4	142.0	187.6	233.2	278.8	324.4	370.0	415.6

Questions

C-1. Based on your graph, what are the values for θ_0 , ω_0 , and α ? Enter these values on the left blanks. What must the units for those variables be in order to stay dimensionally consistent if θ is supposed to be in (rad) and t is in (s)? Enter units in on the right blanks.

θ_0 : _____

ω_0 : _____

α : _____

In the second semester of physics some of you may encounter the following equation.

$$V = V_0 e^{-t/\tau}$$

Here, V is something called potential which has units of volts (V), V_0 is the potential at the time defined to be zero, t is time in seconds (s) and τ is something called the time constant. Below is a set of data that is exponential, which means that you will want to use the exponential trend line with your fit. Create a plot of potential vs time and answer the following questions based on your plot.

Voltage (V)	Time (s)
10.533	0.5
9.041	1.5
6.736	2.5
5.118	3.5
4.311	4.5
3.447	5.5
2.462	6.5
1.956	7.5
1.495	8.5
1.347	9.5
0.990	10.5
0.831	11.5
0.663	12.5
0.501	13.5

C-2. Based on your graph, what are the values for V_0 and τ ? Enter these values on the left blanks. What must be the units for those variables in order to stay dimensionally consistent if V is supposed to be in (V) and t in (s)? Enter those units on the right blanks.

V_0 : _____

τ : _____

C-3. Let us assume the theoretical accepted value of V_0 was supposed to be 12.0 and the theoretical value for τ was supposed to be 4.20. Based on your results from your graph, calculate the percent error in your results. You can use the following formula for that calculation and the rest of the space to show your work.

$$\text{percent error} = \frac{\text{experimental} - \text{theoretical accepted}}{\text{theoretical accepted}} \times 100\%$$

Part D. Decay of a Radioactive Nuclide

Radioactive nuclei undergo a process called decay, during which the nucleus changes its identity by emitting one or more particles or by emitting a photon. During this process, the nucleus may change from one element to another.

One common example is the nucleus of Americium-241 (^{241}Am or Am-241) which is commonly used in household smoke detectors. It decays by the emission of an alpha-particle which has 2 protons and 2 neutrons. The remaining nucleus is now Neptunium-237.

The decay of any particular nucleus is random and unpredictable; however, the rate of decay of a large quantity of the element follows a simple equation characterized by a well-defined “half-life” for the element. The half-life is the time required for one-half of the particles to decay. In the case of Am-241 the half-life is 432.2 years. Below is the table which shows the number of particles measured every 100 years.

Using this data, you will attempt to determine the appropriate equation for the number of nuclei remaining as a function of time.

N	1000	852	726	618	527	499	382	325
t (yrs)	0	100	200	300	400	500	600	700

Make a graph of the data of N as a function of time and answer the questions below.

D-1. Fit a linear function to the graph. Write the best-fit equation below it and use it to calculate the half-life of Am-241. To do so, assume $N(t) = 500$ and solve for t .

D-2. Now fit an exponential function to the graph. Write the equation for the best-fit curve in the space below and use it to calculate the half-life of Am-241. As before, set $N(t) = 500$ and solve for t .

D-3. Now fit a quadratic to the graph. Write the equation for the best-fit curve in the space below and use it to calculate the half life of Am-241. As before, set $N(t) = 500$ and solve for t .

D-4. Calculate the percent error between each of the three estimates for the half-life and the actual half-life of 432.2 years.

Linear fit: _____

Exponential fit: _____

Quadratic fit: _____

Based on these results, which function would you assume best represents the nature of radioactive decay?

D-5. It turns out that the exponential function is the equation that actually describes the decay of a radioactive nuclide. Does this fact coincide with your conclusion above?

Yes No

If No, what may be responsible for the error?

D-6. Go to the plot of the quadratic and exponential fits. Right-click on the fit equation and click “Format Trendline Label.” This will bring up a panel on the right. Click on the drop-down box that says General. Choose Scientific. You will notice that the numbers in your fit equations have changed. For both of these fits, recalculate the half lives and recalculate the percent error between your new calculated results and the actual half-life of 432.2 years. Use the rest of the page to do these calculations, but place your answers on the lines below.

Exponential fit: Half life = _____ Percent Error = _____

Quadratic fit: Half life = _____ Percent Error = _____

Error and Data Analysis - I

Section A: Propagation of Error I - Significant Figures

Identifying Significant Digits

When you make a measurement there are digits where we are certain of their measured value and estimated digits. We typically consider the certain digits and one estimated digit as significant. If a meter-stick measurement gives a length between 23.5 cm and 23.6 cm, but closer to the first then you could your best estimate for the next to be a 3; the measurement, with four significant figures, is 23.53 cm.

When presented with a number you must be able to identify the significant digits. In scientific notation all digits are significant, assuming there are no leading zeroes. For example, 6.8040×10^{-8} has five significant digits.

Beyond scientific notation, the only subtlety is the digit zero; all non-zero digits are always significant. Zeros to the left of the first non-zero digit are not significant. This becomes an issue when there is a decimal expression; a number like 0.00005020 has four significant digits. All those leading zeros are just placeholders; writing this in scientific notation makes it clear: 5.020×10^{-5} . In large numbers without decimal points, all zeros to the right are not significant. For example, 78000 has two significant digits but 78000., with the decimal point, has five; if the first zero is intended to be significant then you should use scientific notation and write 7.80×10^4 .

Multiplication, Division and Single-variable Functions

In a later laboratory exercise, we will discuss a much more precise way to propagate errors. We have standard rules of thumb we use to keep track of the propagation of error using significant digits. When multiplying or dividing numbers or when applying single-variable functions to numbers we ascribe the smallest number of significant digits in the initial numbers to the final result.

In your discussion of one-dimensional kinematics, you will learn (or have learned) that the time of fall for an object dropped from a height h with an acceleration due to gravity of g is $t = \sqrt{2h/g}$. If we with these values for h and g :

$$h = 15.4 \text{ m} \quad \text{and} \quad g = 9.806 \text{ m/s}^2$$

where h has three significant figures and g has four, then we should round our result to three significant digits. Suppose that after punching these values into your calculator, it displays ten digits: $t = 1.772268073 \text{ s}$. Rounding this to three significant digits gives the desired result.

$$t = \sqrt{2h/g} = 1.77 \text{ s}$$

The square root function used here is a function of just one variable, so its presence does not affect the number of digits in the final result. We apply this rule to all single-variable functions.

Addition or Subtraction

Number	Number of Significant Digits	Last Significant Decimal Place
2.0150	5	10^{-4}
28000	2	10^3
1200.	4	10^0
0.0023050	5	10^{-7}

When adding or subtracting numbers, it is not the number of significant figures that matters; it is the last significant decimal place of each number. When adding or subtracting numbers we pay attention to the largest of the last significant decimal place and ignore digits beyond that. Consider the table above. If you are summing all these then the calculator gives 29202.0173, but we should round to the thousands or 10^3 place.

$$2.0150 + 28000 + 1200. + 0.002305 = 29000$$

Adding the first and the last numbers gives a last significant decimal place of one ten-thousandth or 10^{-4} .

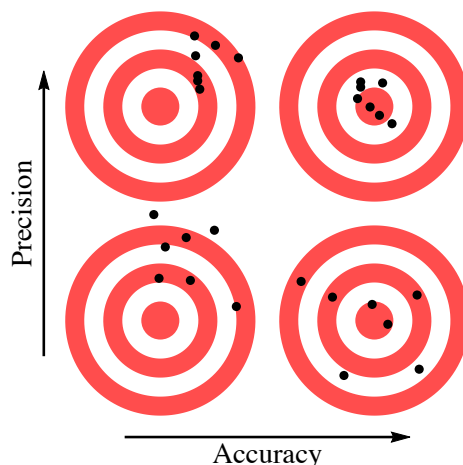
$$2.0150 + 0.002305 = 2.0173$$

Section B: Accuracy, Precision and Types of Error

Accuracy and Precision

The words accuracy and precision are often used interchangeably but they have quite different meanings. Accuracy describes how close your result is to the accepted or theoretical value. Precision describes how close your data is to itself; a smaller spread is more precise.

It is common to illustrate this with a target practice example. The diagram shows distributions of shots at a target.



The distributions that are more tightly packed are more precise and those with the center of the distribution closer to the center of the target are more accurate.

Percent Difference

Percent difference is a simple measure of precision for two experimental values. Label these values x_1 and x_2 . The percent difference is the absolute value of the difference over the average.

$$\text{percent difference} = \frac{|\text{difference}|}{\text{average}} \times 100\% = \frac{|x_2 - x_1|}{\frac{(x_1 + x_2)}{2}} \times 100\%$$

Percent Error

Percent error is a simple measure of accuracy, comparing an experimental value E to its theoretical or accepted value A .

$$\text{percent error} = \frac{\text{experimental} - \text{accepted}}{\text{accepted}} \times 100\% = \frac{E - A}{A} \times 100\%$$

Often percent error is defined with an absolute value. We will use the convention that its sign indicates whether the experimental value is too large or too small.

Systematic Errors

Systematic errors affect accuracy.

As a trivial example, suppose you need to measure a length with a meter stick. If the end of a meter stick had been repeatedly dropped to the floor, then it may be too short and thus read too long. If a measuring device is incorrectly calibrated, then there is a systematic error; every measurement will reflect that poor calibration. If a scale is not properly zeroed, then all measurements are systematically off. Sometimes instruments drift and need to be regularly recalibrated. If an analog meter is read from an angle then that introduces a parallax error; imagine looking at an analog meter, like an analog speedometer, from an angle.

If an important factor is ignored then that introduces error, typically a systematic one. For instance, if the thermal expansion of a ruler or calipers is ignored, then small systematic errors are introduced. Note that if the measurements are made under varying conditions then this error could be random. Ignoring air resistance can introduce systematic error.

Some measurements require a significant lag time that must be respected; for instance, in thermodynamics a system must be sufficiently near equilibrium before a temperature can be measured and that takes time.

Random Errors

Random errors affect precision.

Often there is physical variation in the thing being measured. If one wants to find the mass of a penny then there will be considerable variation in the masses found; there is some small variation in the masses of pennies as they are minted and a much larger variation after being handled.

Any instrument has a finite resolution, and this introduces random errors. Random physical effects like vibration, electronic noise and drafts and temperature fluctuations are often significant.

Section C: Random Errors and Probability Distributions

Probability and Probability Distributions

Randomness is described by probability. Flip a coin and it has a 50% probability landing on either heads or tails. If a second coin is flipped it has the same 50% probability for each outcome; this results in a 50% probability of one head and one tail and a 25% chance for two heads and 25% for two tails. Flipping many coins results in a distribution of probabilities for getting different numbers of heads (or tails.)

By repeating an experiment many times one can reproduce the probability distribution. If for the coin flip, one defines success as a coin landing on heads, then the probability of success is $\frac{1}{2} = 50\%$. Flipping several coins will not result in half successes but as the number of flips becomes large, the number of successes divided by the number of flips will approach $\frac{1}{2}$. This illustrates a fundamental general principle: For any probability distribution, the number of successes over the number of trials will approach the probability of success as the number of trials approaches infinity.

A *probability distribution* is some function that gives probabilities of outcomes for some random variable; the sum of all probabilities must always be one. For a simple example let x be a random variable for a coin flip with $x = 1$ for a head and $x = 0$ for a tail. The probability distribution is then

$$P(0) = \frac{1}{2} = P(1) \text{ where } 1 = P(0) + P(1) .$$

As another example consider the sum of the faces of two dice, which can vary from 2 to 12. Here we get:

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

These two examples are discrete distributions. For a general discrete distribution, we have a random variable where value x has probability $P(x)$. Here $1 = \sum_x P(x)$ where the sum is over all values of x .

The *expectation value* of some function $f(x)$ of a random variable is written $\langle f(x) \rangle$ and is defined as:

$$\langle f(x) \rangle = \sum_x f(x)P(x) \tag{1}$$

In addition to discrete distributions we also have continuous distributions. Here we have a continuous random variable x . The probability of x being between x and $x + dx$ is $P(x)dx$. The condition that the probabilities sum to one becomes the “normalization” condition that: $\int P(x)dx = 1$, where the (definite) integral is over all possible values of the random variable x . The probability that x has a value between x_1 and x_2 is:

$$\int_{x_1}^{x_2} P(x)dx .$$

In the continuous case, the expectation value becomes

$$\langle f(x) \rangle = \int f(x)P(x)dx ,$$

where again the integral is over all possible values of x .

For both discrete and continuous distributions, the mean μ and standard deviation σ are defined as

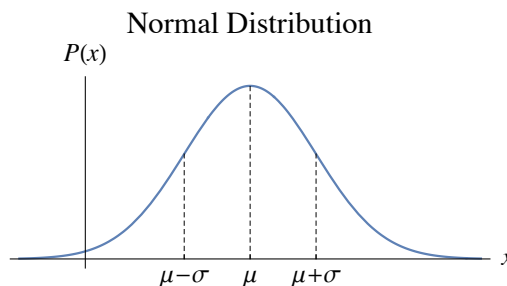
$$\mu = \langle x \rangle \text{ (mean) and } \sigma = \sqrt{\langle (x - \mu)^2 \rangle} = \sqrt{\langle x^2 \rangle - \mu^2} \text{ (standard deviation)}$$

The standard deviation is a measure of the spread of the distribution; a small standard deviation means the highest probability is that the random variable is close to the mean and a large one implies a larger spread. The square of the standard deviation σ^2 is known as the variance.

The most important example of a continuous distribution is the *normal distribution*.

$$P_{\mu,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \text{ where } \int_{-\infty}^{\infty} P_{\mu,\sigma}(x)dx = 1$$

The factor multiplying the exponential is there to make it normalized, i.e. integrating to one. By evaluating the appropriate integrals, one can show that the μ and σ in the expression are the mean and standard deviation of the distribution.



This is just what is popularly referred to as a “bell-shaped” curve. This shape is also referred to as a Gaussian. Integrating Gaussians requires the definition of a new function as its antiderivative, the *error function*.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Using this we can find the probability that a normal distribution lies between x_1 and x_2 as

$$\int_{x_1}^{x_2} P_{\mu,\sigma}(x) dx = \frac{1}{2} \operatorname{erf}\left(\frac{x_2 - \mu}{\sqrt{2} \sigma}\right) - \frac{1}{2} \operatorname{erf}\left(\frac{x_1 - \mu}{\sqrt{2} \sigma}\right)$$

Independent Random Variables

Two random variables x and y are independent when they are not correlated; this means that $P(x, y) = P(x) P(y)$, where $P(x, y)$ is the probability of both x and y occurring. As an example, smoking cigarettes and lung cancer are not independent, because they are significantly correlated, but left-handedness and lung cancer are most likely independent. For independent random variables $\langle xy \rangle = \langle x \rangle \langle y \rangle$. This is easy to show for discrete distributions.

$$\langle xy \rangle = \sum_{x,y} xy P(x, y) = \sum_x \sum_y xy P(x) P(y) = \left(\sum_x x P(x) \right) \left(\sum_y y P(y) \right) = \langle x \rangle \langle y \rangle$$

It is an important fact that the variance of the sum of two independent random variables is the sum of the variances.

$$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$$

This will be proved in the Appendix.

The mean of the sum is also the sum of the means.

$$\mu_{x+y} = \langle x + y \rangle = \langle x \rangle + \langle y \rangle = \mu_x + \mu_y$$

It is an important fact that when many of different or the same probability distributions are added (or averaged) the result tends toward a normal distribution with the same mean and standard deviation. This is known as the *Central Limit Theorem*. Random errors are usually the cause by the accumulation of many small effects; because of this they are modeled well by normal distributions.

Section D: Data Analysis I - Mean and Standard Deviation

Data Analysis

Suppose you have randomly distributed data. x_i is a random data point with a continuous distribution with the same probability $P(x)$, mean μ and variance σ^2 . Suppose there are N such data points in the data set: $\{x_i\} = \{x_1, \dots, x_N\}$. The *statistical mean*, \bar{x} , of the data set is the average of the data points.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = \frac{1}{N} \sum_i x_i = \frac{x_1 + x_2 + \dots + x_N}{N}$$

The statistical mean is an estimate of the mean of the probability distribution; its expectation value is the mean of the distribution: $\langle \bar{x} \rangle = \mu$.

The statistical standard deviation s (or s_x to be more explicit) is an estimate of σ , the standard deviation of the distribution. The statistical standard deviation of the same data set $\{x_i\}$ is defined in terms of the statistical mean \bar{x} .

$$s = \sqrt{\frac{1}{N-1} \sum_i (x_i - \bar{x})^2} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_N - \bar{x})^2}{N-1}}$$

The factor of $N-1$ seems odd, but it is needed to guarantee that the expectation of the s^2 is σ^2 , the variance of the probability distribution: $\langle s^2 \rangle = \sigma^2$. We will not prove this here.

The statistical standard deviation is a measure of precision and will not reflect systematic errors.

Appendix: Data Analysis I - Mean and Standard Deviation

Proof of the Variance of the Sum

For independent random variables x and y , $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$. This is not a difficult proof. Evaluate the two terms separately in $\sigma_{x+y}^2 = \langle (x+y)^2 \rangle - \mu_{x+y}^2$.

$$\langle (x+y)^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2\langle xy \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2\langle x \rangle \langle y \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2\mu_x \mu_y$$

Here we have used $\langle xy \rangle = \langle x \rangle \langle y \rangle$ for independent x and y . Using $\mu_{x+y} = \mu_x + \mu_y$ we can evaluate the second term.

$$\mu_{x+y}^2 = (\mu_x + \mu_y)^2 = \mu_x^2 + \mu_y^2 + 2\mu_x \mu_y$$

Subtracting cancels the $2\mu_x \mu_y$ terms, giving the desired result.

$$\sigma_{x+y}^2 = \langle (x+y)^2 \rangle - \mu_{x+y}^2 = \langle x^2 \rangle - \mu_x^2 + \langle y^2 \rangle - \mu_y^2 = \sigma_x^2 + \sigma_y^2$$

Worksheet: Error and Data Analysis - I

Equipment and Setup: Mathematica file – DataAnalysis-I.nb

Section A: Significant Figures

Questions

A-1. Complete the table showing the number of significant digits and the last significant decimal place for each number given.

Number	Number of Significant Digits	Last Significant Decimal Place
3.14159		
590		
590.		
0.0003854		
2.6×10^{21}		

A-2. Give your answers with appropriate significant figures.

$$3.14159 + 0.0003854 = \underline{\hspace{2cm}}$$

$$3.14159 \times 0.0003854 = \underline{\hspace{2cm}}$$

A-3. Using $A = 0.02528$ m, $\omega = 364.6$ s⁻¹ and $t = 0.00243$ s, calculate the result with appropriate significant figures and units. (Note that to evaluate such a function it is essential to use radians, so put your calculator in its radians mode.)

$$A \sin(\omega t) = \underline{\hspace{2cm}}$$

Section B: Target Practice

Computer Setup for Section B

1. The interactive panel shows a target. At the top are controls: there is a Setter Bar with four buttons, one for each trial.
2. Start with Trial 1. Click on the panel to get cross-hairs for aiming. Drag the cross-hairs to the center.
3. Click “Fire” and a dot will appear. Repeat this many times to observe a pattern.

4. Repeat the procedure for trials 2 through 4.
5. The small button in the upper right resets the panel.

Data Recording for Section B

1. Rank the four trials for accuracy and precision in the table below. (Write 1 through 4 into the appropriate boxes.)

Precision ↑		
	→ Accuracy	

Questions

B-1. List some factors that could account for a lack of precision in this simulation of target practice.

B-2. List some factors that could account for a lack of accuracy in this simulation.

Section C: Probability Distributions

Computer Setup for Section C.1

1. The interactive panel shows a Setter Bar with two options: “Coin Flip of Heads” and “Dice Rolls of One”.
2. Below that is a button “Random” that generates the coin flips or dice rolls. Next to the button there is an Input Field for n , the number of coins or dice. n must be a positive integer.
3. The small button in the upper right resets the panel.

Data Recording for Section C.1

1. Start with coin flips and set $n = 10$. For three different trials record the number of heads, x , and the fraction of heads, x/n .

Trial	1	2	3
x			
x/n			

2. Now set $n = 1000$ and similarly record the number of heads, x , and the fraction of heads, x/n for three trials.

Trial	1	2	3
x			
x/n			

3. Now do the same for $n = 1,000,000$.

Trial	1	2	3
x			
x/n			

4. Change the Setter Bar to dice rolls and set $n = 10$. For three different trials record the number of ones, x , and the fraction of ones, x/n .

Trial	1	2	3
x			
x/n			

5. Repeat with $n = 1000$

Trial	1	2	3
x			
x/n			

6. And for $n = 1,000,000$.

Trial	1	2	3
x			
x/n			

Questions

C-1. The probability of a coin flipping to heads is $1/2$. Discuss what happens to x/n as n increases in steps 1 through 3.

C-2. The probability of a die rolling one is $1/6$. Discuss what happens to x/n as n increases in steps 4 through 6.

Computer Setup for Section C.2

1. This simple interactive panel has two sliders for the mean μ and the standard deviation σ . The reset button is at the upper right.

Data Recording for Section C.2

1. Adjust the slider for the mean μ . What happens to the graph as μ increases?

2. Adjust the slider for the standard deviation σ . What happens to the graph as σ increases?

Computer Setup for Section C.3

1. There is an Interval Slider that varies the values of x_1 and x_2 . When at the edge of the Interval Slider the values are set to $\pm\infty$.
2. The area between these x -values is listed to the right.
3. A small reset button is at the upper right.

Data Recording for Section C.3

1. For data from a normal distribution, what fraction of the values are less than the mean?

2. What fraction of the values are within one standard deviation σ of the mean? 2σ ? 3σ ?
3. In particle physics they use the 5σ standard. When they are certain of a result within five standard deviations, they are ready to publish their discovery. What is the probability of a normal distribution being more than 5σ from the mean?

Computer Setup for Section C.4

1. There are several controls at the top. At the left there are three tabs: Flip Coins, Binomial Distribution and Sum of Dice Roll. Below each tab are pop-up menus specific to that tab. For each n is the number of coins, trials or dice. For the Binomial Distribution p is the probability of success, where the coin flip is the special case of $p = \frac{1}{2}$.
2. To the right is a Setter Bar, where you choose either a graph of the discrete probability distribution, that same graph with the normal distribution (Gaussian) with the same standard deviation and mean, or a probability table for the distribution.
3. The reset button is at the upper right.

Data Recording for Section C.4

1. What is the probability of flipping 50 coins with exactly 25 heads? (Hint: read this directly off the table.)
2. The variance (the square of the standard deviation σ) for a binomial distribution is $\sigma^2 = n p (1 - p)$. Find the standard deviation for $n = 25$ and $p = 0.8$, and compare with the result from the interactive panel. Now do the same for $n = 60$ and $p = 0.4$.
3. Now click Graph with Gaussian on the Setter Bar. Comment on the comparison between the discrete distribution and the Gaussian as n gets larger.

Section D: Data Analysis**Computer Setup for Section D**

1. The interactive panel generates a random set of data.
2. Clicking “Generate Data” selects a value of μ and σ , the mean and standard deviation of a normal distribution. It then generates 5 random points with that distribution.
3. The reset button is at the upper right.

Data Recording for Section D

1. Click “Generate Data” and record the mean, standard deviation and the data.

$\mu =$ _____	x
$\sigma =$ _____	

Questions

D-1. Calculate the statistical mean \bar{x} and the statistical standard deviation s for this data. Show all work.

D-2. The statistical mean \bar{x} and the statistical standard deviation s can be viewed as approximations to μ and σ , the mean and standard deviation of the distribution. Calculate the percent errors in \bar{x} and s taking μ and σ as the accepted values.

% error $\bar{x} =$ _____

% error $s =$ _____

Error and Data Analysis - II

Section A: Mathematical Preliminary – Partial Derivatives and the Total Differential

Partial Derivatives

In Calculus I, the derivative of functions of one variable was discussed; using the rules of differentiation (sum rule, product rule, quotient rule, chain rule, etc.) you should now be able to evaluate the derivative of any function built up from known functions. Calculus I and also Calculus II discuss only single variable calculus. All of multivariable calculus is put off to the third course, Calculus III. The key subtlety of multivariable calculus is integration. We will now see that taking derivatives of functions of more than one variable, partial differentiation, is no more difficult than the ordinary derivatives learned in Calculus I.

If $f(x)$ is a function of one variable then we write its derivative, the *ordinary derivative*, as $\frac{df}{dx}$. For functions of more than one variable, we use *partial derivatives*. For notation, we replace the d with ∂ and then we write the partial derivatives of $f(x, y)$ as $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

Suppose you have a function $f(x, y, \dots)$ of several independent variables, x, y, \dots . To evaluate a partial derivative with respect to one independent variable you just take the ordinary derivative with respect to that variable, while treating the other independent variables as constants. As an example, consider

$$f(x, y) = axy^2 - bx^3$$

Where x and y are the independent variables and a and b are constants. The partial derivatives become

$$\frac{\partial f}{\partial x} = ay^2 - 3bx^2 \quad \text{and} \quad \frac{\partial f}{\partial y} = 2axy.$$

Total Differential

The total differential of some function can be interpreted as the infinitesimal change in the function due to infinitesimal changes in each of its independent variables. First, consider a function of one variable $f(x)$. If x changes by the infinitesimal amount dx , then $f(x)$ will shift by $(df/dx)dx$. The total differential is written

$$df = \frac{df}{dx} dx.$$

For functions of more than one variable $f(x, y, \dots)$ we write the total differential with partial derivatives

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \dots.$$

We now find the total differential for the previous example function.

$$f(x, y) = axy^2 - bx^3 \quad \Rightarrow \quad df = (ay^2 - 3bx^2) dx + 2axy dy$$

Section B: Propagation of Error II - General Functions of Random Variables

Uncertainty and Notation

In the earlier experiment, Error and Data Analysis – I, we discussed the mean and standard deviation for a probability distribution and for a statistical sample of data. For some variable x we will take its uncertainty δx , to be the statistical standard deviation of a data, an estimate of the error in x or a combination of both.

The standard notation for writing such uncertainty is to use the \pm notation. A value x with uncertainty δx is written as $x \pm \delta x$. An alternative notation, which is common when listing values of constants, is to write digits as $xxxx(x)$ or $xxxx(xx)$. The expression in brackets gives the uncertainty in the last one or two digits, respectively. For example, consider the listed values of the Earth's mass

$$5.9722(6) \times 10^{24} \text{kg} \Leftrightarrow (5.9722 \pm 0.0006) \times 10^{24} \text{kg}$$

and the electron's mass.

$$9.1093837015(28) \times 10^{-31} \text{kg} \Leftrightarrow (9.1093837015 \pm 0.0000000028) \times 10^{-31} \text{kg}$$

Propagation of Uncertainty

In the earlier experiment, we discussed the mean, variance and standard deviation for a probability distribution. The key thing we need from that discussion is that the variance (the square of the standard deviation) of the sum of two (or more) independent random variables is the sum of the variances.

$$\sigma_{x+y+\dots}^2 = \sigma_x^2 + \sigma_y^2 + \dots$$

We will consider the uncertainty δx to our best estimate of σ_x , the standard deviation of the underlying probability distribution for x . It follows from this, that for independent random variables x and y with uncertainties δx and δy , that the uncertainty in the sum is

$$\delta(x + y + \dots) = \sqrt{\delta x^2 + \delta y^2 + \dots}.$$

We now want to general this for arbitrary function of multiple independent random variables. We make the assumption that uncertainties are small, so they may be viewed as infinitesimal. The total differential describes the infinitesimal change in f due to the infinitesimal changes in variables.

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \dots$$

When x and y are independent, then each term in the above sum is independent.

It follows that the uncertainty in the function f is given by

$$\delta f(x, y, \dots) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y^2 + \dots} \quad (1)$$

Special Case: Function of One Variable

For a function of one variable the partial derivatives become a single ordinary derivative and the square-root gives an absolute value.

$$\delta f(x) = \left| \frac{df}{dx} \right| \delta x \quad (2)$$

As an example of this let us find $u = \sin \theta$, where $\theta \pm \delta \theta = 58.32^\circ \pm 0.07^\circ$. First find the uncertainty. (Note that angles are dimensionless quantities and here the $\delta \theta$ must be converted to radians using $1^\circ = \frac{\pi}{180}$.)

$$\delta u = \left| \frac{d \sin \theta}{d \theta} \right| \delta \theta = |\cos \theta| \delta \theta = |\cos 58.32^\circ| \times 0.07 \frac{\pi}{180} = 0.00064$$

Using $\sin 58.32^\circ = 0.85099$, we get:

$$\sin(58.32^\circ \pm 0.07^\circ) = 0.8510 \pm 0.0006$$

Special Case: Products, Quotients and Powers

Now we will consider a function of the form

$$u = f(x, y, \dots) = a \cdot x^m \cdot y^n \cdot \dots$$

Where a is a constant and the powers m_i can have any real value. Note that you can use negative powers for division. There is a standard trick that is used here that involves “logarithmic derivatives”. Take the natural log of both sides and then evaluate the total differential.

$$\ln(u) = \ln(a) + m \ln(x) + n \ln(y) + \dots$$

Taking the differential of a logarithm gives a simple result: $d \ln(u) = \frac{du}{u}$. The total differential of the expression then gives

$$\frac{du}{u} = m \frac{dx}{x} + n \frac{dy}{y} + \dots$$

This allows us to read off the partial derivatives

$$\frac{\partial u}{\partial x} = m \frac{u}{x}$$

and then write an expression for the uncertainty.

$$\frac{\delta u}{|u|} = \sqrt{m^2 \left(\frac{\delta x}{x}\right)^2 + n^2 \left(\frac{\delta y}{y}\right)^2 + \dots} \quad (3)$$

There is a simple interpretation of this. The relative uncertainty of a variable x is $\frac{\delta x}{x}$. Significant figures are a crude measure of relative uncertainty. We can see that the variable with the largest relative uncertainty has the largest effect on the relative uncertainty of the result.

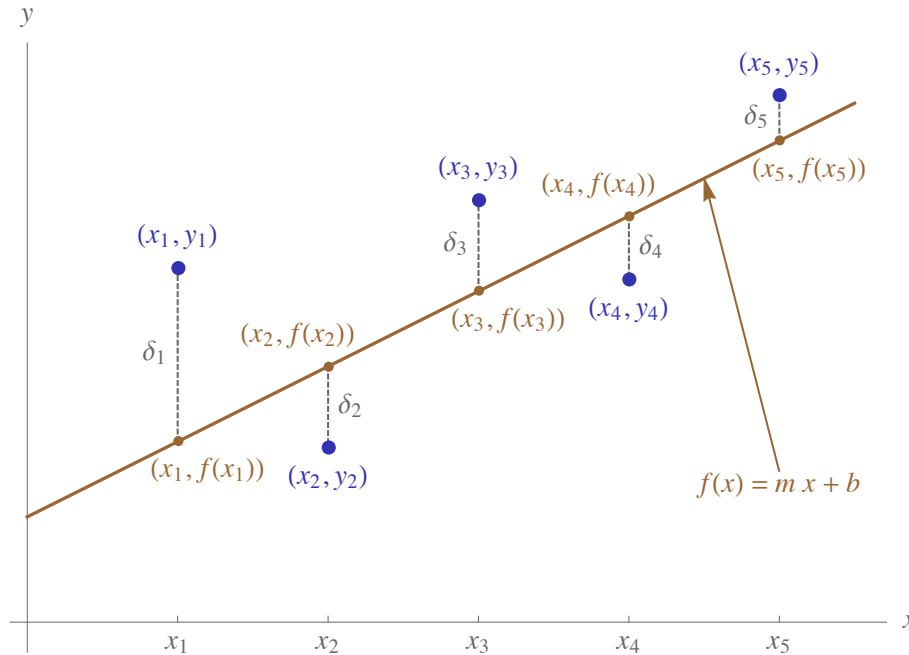
Section C: Data Analysis II – The Method of Least Squares

Data and Curve Fitting

We will consider a set of data points $\{(x_1, y_1), (x_2, y_2), \dots\}$ and a function $y = f(x)$ that fits the data. Generally, the function $f(x)$ will have some parameters and we find the optimal set of parameters to get a best-fit. Here, we will consider the simplest case of a best-fit line. The function has the form:

$$y = f(x) = m x + b$$

and the parameters are m and b . We will use a method known as *least squares* to find the optimal values of m and b and thus the best-fit line.



The error for a particular data point (x_i, y_i) is $\delta_i = |f(x_i) - y_i|$. The idea of the least squares method is to sum the squared errors and then minimize this sum over the parameters m and b . Label the sum of these squared errors as $S(m, b)$.

$$S(m, b) = \sum_i \delta_i^2 = \sum_i (f(x_i) - y_i)^2 = \sum_i (m x_i + b - y_i)^2$$

Each sum goes from 1 to N , where N is the number of data pairs.

Finding the Parameters

In Calculus I you learned that to find a local maximum or minimum of a function, set the derivative to zero. Here since $S(m, b)$ is a function of two variables, we will minimize it by setting both partial derivatives to zero.

$$0 = \frac{\partial}{\partial m} S(m, b) = 2 \sum_i (m x_i + b - y_i) x_i = 2 \left(m \sum_i x_i^2 + b \sum_i x_i - \sum_i x_i y_i \right)$$

$$0 = \frac{\partial}{\partial b} S(m, b) = 2 \sum_i (m x_i + b - y_i) = 2 \left(m \sum_i x_i + N b - \sum_i y_i \right)$$

The reason for the N in the second expression is that the sum of a constant gives that constant multiplied by number of data pairs N , $\sum b = Nb$. Dividing a sum by N gives an average.

$$\bar{x} = \frac{1}{N} \sum_i x_i, \quad \bar{y} = \frac{1}{N} \sum_i y_i, \quad \overline{x^2} = \frac{1}{N} \sum_i x_i^2 \quad \text{and} \quad \overline{xy} = \frac{1}{N} \sum_i x_i y_i$$

Dividing the two expressions (from the derivatives) by $2N$ gives:

$$0 = m \overline{x^2} + b \bar{x} - \overline{xy} \quad \text{and} \quad 0 = m \bar{x} + b - \bar{y}.$$

To eliminate b multiply the second expression by \bar{x} and subtract it from the first expression.

$$0 = m (\overline{x^2} - \bar{x}^2) - (\overline{xy} - \bar{x}\bar{y}).$$

With this we can find m and using the second expression we can solve for b in terms of m .

$$m = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} \quad \text{and} \quad b = \bar{y} - m\bar{x}$$

Let us summarize with an outline of the procedure for finding the best-fit line. Start with the data $\{(x_1, y_1), (x_2, y_2), \dots\}$, next evaluate the averages \bar{x} , \bar{y} , $\overline{x^2}$ and \overline{xy} , and then find m and b .

Quality of Fit – R^2

The smaller the value of $S(m, b) = \sum_i \delta_i^2$, the better the fit. This value has the units of the y -values and rescales with the y -values; if we multiply all the y -values by 100 then the $S(m, b)$ value increases by a factor 100^2 . We want a dimensionless (without units) and scale invariant measure of quality of fit. From our data set we can define D by:

$$D = \sum_i (y_i - \bar{y})^2$$

Using D as a denominator we can get $S(m, b)/D$ as dimensionless and scale-invariant. With S/D , a smaller value is a better fit. We define R^2 to vary typically from 0 to 1 with a perfect fit being $R^2 = 1$, but negative values are possible

$$R^2 = 1 - \frac{S(m, b)}{D} = 1 - \frac{\sum_i \delta_i^2}{\sum_i (y_i - \bar{y})^2}$$

Worksheet: Error and Data Analysis - II

Equipment and Setup: Mathematica file – DataAnalysis-II.nb

Section A: Partial Derivatives and the Total Derivative

Computer Setup for Section A

- The interactive panel shows a 3-dimensional plot of $z = f(x, y) = 0.02x^2y + 0.06xy - 0.06y^2 + 1.2$. Note that the x , y and z directions are labeled.
- The control is a 2-dimensional slider that varies the x and y coordinates.
- The small button in the upper right resets the panel. The coordinates are reset to their initial values of $x = -2$ and $y = 2$.
- The red dot is the point $P = (x, y, z)$, where $z = f(x, y)$. The blue plane is the surface of constant x passing through P and the green plane is the surface of constant y through P . Curves of constant x and y are drawn onto these planes.
- At the right side these two planes are redrawn as graphs of z versus y and z versus x , with colors that match the planes. The curves are also drawn on the side graphs and the line tangent to P is also included, which has a slope corresponding to the partial derivatives. The numerical values of the partial derivatives at P are included above the graphs.

Data Recording for Section A

- Move the slider to find some position where both partial derivatives are positive, $\partial f/\partial x > 0$ and $\partial f/\partial y > 0$. Record the values of (x, y) , (There are many possible answers but find some position.)

$x =$ _____ and $y =$ _____

- Do the same for, $\partial f/\partial x < 0$ and $\partial f/\partial y < 0$.

$x =$ _____ and $y =$ _____

- Hit the reset button to return to the initial values of $x = -2$ and $y = 2$ and record the values of the partial derivatives at that position. You will calculate these values later in a question.

$\frac{\partial f}{\partial x} =$ _____ and $\frac{\partial f}{\partial y} =$ _____

Questions

A-1. Follow the discussion in the theory section to evaluate the partial derivatives of the function below, as functions of x and y and also find its total differential.

$$f(x, y) = ax^4 + bxy^2 - cy$$

$$\frac{\partial f}{\partial x} = \underline{\hspace{10em}}, \quad \frac{\partial f}{\partial y} = \underline{\hspace{10em}}$$

$$\text{and } df = \underline{\hspace{10em}}$$

A-2. Do the same for the function used in the interactive panel:

$$f(x, y) = 0.02x^2y + 0.06xy - 0.06y^2 + 1.2$$

$$\frac{\partial f}{\partial x} = \underline{\hspace{10em}}, \quad \frac{\partial f}{\partial y} = \underline{\hspace{10em}}$$

$$\text{and } df = \underline{\hspace{10em}}$$

A-3. Now evaluate these partial derivatives at $x = -2$ and $y = 2$ and verify that they are the same as you recorded in part 3 of the data above.

$$\frac{\partial f}{\partial x} = \underline{\hspace{10em}}, \quad \frac{\partial f}{\partial y} = \underline{\hspace{10em}} \quad \text{and } df = \underline{\hspace{10em}}$$

Section B: Propagation of Error

Questions

B-1. Suppose that $y = \ln(x)$, where $x = 1.0385 \pm 0.0003$. Find y with the appropriate uncertainty. Use equation (2).

B-2. Consider $y = A \sin(\omega t)$, which is a function of A , ω and t , where $A = (2.054 \pm 0.012)\text{m}$,
 $\omega = (2735 \pm 8) \text{ s}^{-1}$ and $t = (0.000341 \pm 0.000015)\text{s}$. Find y with the appropriate uncertainty. Use the general expression, equation (1).

B-3. In simple kinematics, the time of fall for an object dropped from a height h is $t = \sqrt{2h/g}$. Suppose the values of g and h where experimentally determined to have the values and uncertainty

$$g = (9.8066 \pm 0.0021)\text{m/s}^2 \text{ and } h = (25.631 \pm 0.015)\text{m}.$$

B-4. Find $\delta t/t$, the relative uncertainty in t and also find the t value with its uncertainty. Use equation (3).

Section C: The Method of Least Squares**Computer Setup for Section C**

1. When the “Generate Data” button is pressed, the interactive panel generates a set of 7 random (x, y) data points. We are trying to find the best-fit line for this data.
2. There are two sliders that vary the values of the slope m and intercept b of a line; the values of m and b are shown to the right of each slider.
3. The method of least squares minimizes $\sum_i \delta_i^2$, the squared error discussed in the theory section. There is a check box “Show Squared Error and R^2 ”, where R^2 is the quality of fit parameter also discussed. Checking the box gives these values below. The “Snap to Best-fit” check box will be used at the end.
4. The small button in the upper right resets the panel.

Data Recording for Section C

1. Press the “Generate Data” button and record the values.

x	y

2. Adjust the sliders for the slope and intercept to get what, by your eye, is the best-fit. You will have to alternate varying m and b to get your best values. Record these values.

$$m = \underline{\hspace{2cm}} \quad \text{and} \quad b = \underline{\hspace{2cm}}$$

3. Now check the “Show Squared Error and R^2 ” box and record their values from your by-eye estimate.

$$\text{Squared Error} = \underline{\hspace{2cm}} \quad \text{and} \quad R^2 = \underline{\hspace{2cm}}$$

4. Repeat the procedure from part 1, varying the sliders, to get values of m and b that give the smallest squared error and largest R^2 . (This is a bit tedious and sensitive to the controls. You have to make small adjustments, alternating between both sliders. You do not have to do this perfectly.) Record m and b to and also the squared error and R^2 values.

$$m = \underline{\hspace{2cm}} \quad \text{and} \quad b = \underline{\hspace{2cm}}$$

$$\text{Squared Error} = \underline{\hspace{2cm}} \quad \text{and} \quad R^2 = \underline{\hspace{2cm}}$$

5. Now check “Snap to best-fit”. This changes the values of m and b and the squared error and R^2 values. Record these values. You will calculate these m and b values in the question that follows.

$m =$ _____ and $b =$ _____

Squared Error = _____ and $R^2 =$ _____

Questions

C-1. Use the table below to guide you through the calculation of the m and b values. The x and y columns come from your data table. The xy and x^2 columns are evaluated from the x and y values to the left. At the bottom, for the row with summations, you sum the numbers above. For the row with averages, you divide the sums by n , the number of data points (7 in this case) to get the averages.

x	y	xy	x^2
$\sum_i x_i =$	$\sum_i y_i =$	$\sum_i x_i y_i =$	$\sum_i x_i^2 =$
$\bar{x} =$	$\bar{y} =$	$\overline{xy} =$	$\overline{x^2} =$

$m = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} =$ _____ and $b = \bar{y} - m\bar{x} =$ _____

C-2. Your values should be the same as from part 5. Are they?

Free Fall Experiment

Equipment and Setup: Picket fence, Photogate, Capstone file – Free Fall.cap

Theory

Free fall is an example of one-dimensional motion with constant acceleration. When under the influence of only gravity, meaning there is no friction or any other force, bodies will fall with a constant downward acceleration of g , the accepted value of g is:

$$g_{\text{accepted}} = 9.80 \frac{\text{m}}{\text{s}^2}$$

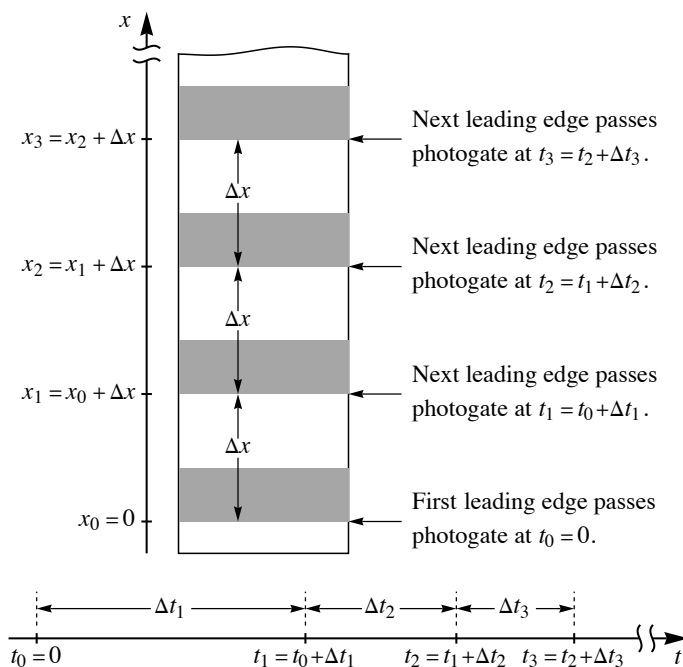
In this experiment we will measure g . The tools we will use to measure acceleration will be repeated in several later experiments. One of the goals of this experiment is to understand how the Capstone program calculates velocity from a falling object that is referred to as a picket fence.

The Picket Fence and the Photogate

A photogate uses a photocell with an infrared light source and detects whether the light beam is blocked or not. The picket fence we will use is a clear strip of plastic with black stripes. When the picket fence is dropped through the photogate, the Capstone program records when the light is blocked. Specifically, it is measuring the time between each time the light beam is blocked; this is when the leading edge of the black stripes passes through the photogate. The distance between leading edges is a constant Δx , which you will measure. The times between leading edges: $\Delta t_1, \Delta t_2, \dots$ get smaller with time because the picket fence is speeding up.

A running total of times t_0, t_1, t_2, \dots can then be found, where we start with $t_0 = 0$. By adding the next Δt_i to the previous time give the next time $t_i = t_{i-1} + \Delta t_i$

Getting a running total of x values is similar, but easier because the Δx is constant; start with $x_0 = 0$ and add the Δx to the last x to get the next x value: $x_i = x_{i-1} + \Delta x$. Thus, we can get position as a function of time from the data. This is shown in the diagram to the right and above.



Velocity and Time Calculation

In addition to position as a function of time, the Δt data combined with the measured value of Δx also gives a way to get velocity as a function of time. This is a bit more subtle. For motion with constant acceleration, we will see that the average velocity over some time interval is the average of the initial and final velocities for that interval. With constant acceleration, the velocity as a function of time graph is a straight line, since $v(t) = v_0 + at$. We also know that since velocity is the time derivative of position x , it follows that the displacement Δx is the area under the velocity versus time graph. The plot below shows this. Because the graph is a line, the area under the graph is the area of a trapezoid. The area of a trapezoid is the same as the area of a rectangle with the same base b , with a height that is the average of the trapezoid's heights h_1 and h_2 .

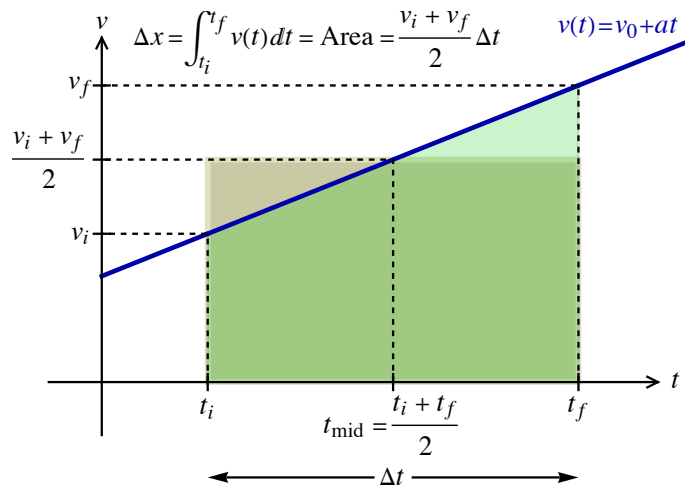
$$\text{Area} = b \frac{h_1 + h_2}{2} \Rightarrow \text{Area} = \Delta x = \Delta t \frac{v_i + v_f}{2}$$

With this it follows that, for constant acceleration, the average velocity is the average of the initial and final velocities.

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{v_i + v_f}{2}$$

This shows how to get the velocity as a function of time from the raw Δt data. Associate the average velocity $\Delta x/\Delta t$ with the instantaneous velocity at $t = t_{\text{mid}}$, the midpoint of the time interval.

$$t = t_{\text{mid}} = \frac{1}{2}(t_i + t_f) \Rightarrow v = v_{\text{ave}} = \frac{\Delta x}{\Delta t}$$



The discussion in the previous section showed how to get a running total of times from the Δt data. For each time interval we find t_{mid} and v_{ave} using the expressions above, remembering that the measured Δx is constant. That then gives the experimental calculation of velocity as a function of time.

Statistics

As a follow-up to the previous experiments on data analysis, we will use calculate the mean and standard deviation of your experimental measurements of the acceleration g . To review the relevant formulas, if you have a list of N experimental values x_i , where in your case the x values are the g values. The mean or average \bar{x} is given by:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

The percent error is a measure of accuracy, comparing the experimental value to the accepted value. It is given by:

$$\% \text{ Error} = \frac{\text{experimental} - \text{accepted}}{\text{accepted}} \times 100\%$$

The standard deviation sd is a measure of precision. It measures the spread of the data or how close the data is to itself; the expression for standard deviation is:

$$sd = \sqrt{\frac{1}{N - 1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

Procedure

Setup

1. Turn on the Pasco interface at your lab station and plug the photogate into Digital Channel 1.
2. In the “Physics Folder” on your computer desktop open Free Fall.cap .
3. The photogate needs to be placed near the edge of the lab table so the picket fence can fall freely through it without hitting the photogate or table. Have something soft under the picket fence so it doesn’t crash into the hard floor. It could crack or break.

(A) Data Recording – Finding Acceleration

1. Hold the picket fence vertically slightly above the photogate. In *Capstone*, click **Record** and drop the picket fence. It should not hit anything or rotate while passing through the photogate. If it falls through on an angle that will introduce error. After the picket fence has passed through the photogate, click the **Stop** button.
2. The times, Δt between leading edges should be listed in the table in *Capstone*. The program automatically makes a plot of velocity versus time. The run number is listed in the information box on the graph.
3. Click on the graph to make it active. Maximize the graph by clicking the “scale axes” tool (above graph, on left). If the data does not look linear then there was some error in dropping the picket fence; discard the and try again. To delete the last run click “Delete Last Run” at the bottom of the display (triangle with X), or delete specific runs using the drop down menu next to this control. You can switch between runs using the drop down menu on “Data Selector” tool above the graph (triangle).
4. Once you have a linear set of data, fit the data with a linear best-fit using the drop down menu on the “Curve Fits” tool above the graph.
5. From the equation displayed as the best-fit line, determine the acceleration of your picket fence and record it in Table 1 below as your measured acceleration for Trial 1.
6. Repeat this process until you have 8 good data runs and have recorded 8 measured values of acceleration on table 1. Delete any unwanted runs, but keep at least 8 good runs.
4. For the eight measured free-fall accelerations, calculate the average, the percent error of that average value using the given accepted value and then the standard deviation or your eight values. Record these in Table A.

(B) Data Recording – Reproducing the Computer Calculation of Velocity versus Time

1. In *Capstone*, review your eight data sets and find the data set that gives the best agreement between the measured acceleration and the accepted value of the acceleration due to gravity, g . Select this data set using the dropdown menu on the “Data Selector” tool above the graph (triangle).
2. Use a ruler to measure the spacing between successive pairs of leading edges of the black stripes on your picket fence. Record that as Δx above Table B.
3. Copy the Δt values of your selected run from the table in *Capstone* into the first column in Table B.
4. Next fill out the column in Table B called ‘Time at Each Leading Edge’. The first value is $t = 0$ s by default. Successively add Δt from the first column to get the next value. (See the instructions below the table and the discussion in the theory section.)
5. Now fill in the column called ‘Midpoint of Time Interval’. This is the average of the two times listed in the adjacent cells to the left.
6. Now fill in the column called ‘Average Velocity for Time Interval’ using the formula given below the table. Note that Δx is the same for each of these calculations, so this should be easy.
7. Now fill in the column called ‘Position of Each Leading Edge’. The first value is 0 m, then add Δx successively to get each value.

(C) Questions and Graphs

1. Question 1 will have you plot position as a function of time from your calculations in Table B and answer questions from that plot.
2. Question 2 will similarly have you plot velocity as a function of time from your calculations in Table B and then answer questions from that plot.

Free Fall – Data Sheet

Name _____ Group _____

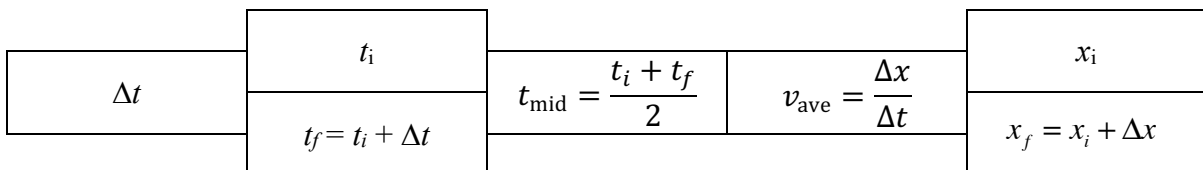
Table A

Trial	Acceleration (m/s ²)		
1			
2			
3			
4			
5			
6		Average: \bar{x}	
7		% Error of Average	
8		Standard Deviation: sd	

Δx is the spacing between the leading edges. $\Delta x =$ _____

Table B

Raw Data <i>(from computer)</i>	Time at each Leading Edge	Midpoint of Time Interval	Average Velocity for Time Interval	Position of Each Leading Edge
Δt (s)	t (s)	t_{mid} (s)	v_{ave} (m/s)	x (m)
	0			0

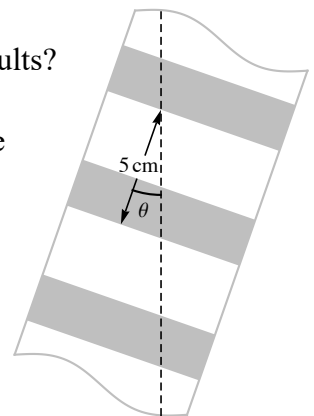


Question 1: Include a graph made using software of x vs. t (from *your* calculations in the Table B. This is from the x column and the t column.) The graph should be a parabola. From the equation of the best-fit parabola find the acceleration and the initial velocity.

Question 2: Also include a graph v vs. t (from *your* calculations of v_{ave} and $t_{\text{mid},}$.) draw the best-fit line and find its slope. Compare the slope with the accepted value of 9.80 m/s^2 ; give the % error.

Question 3: From your graph of v vs. t find the speed of the picket fence when the first leading edge passed the photogate.

Question 4: If the picket fence were dropped at an angle how would this affect your results? (The picket fence is in a vertical plane but its side is skewed from vertical, as shown.) Would your measured value of g be too large or too small? What percent error would be caused by dropping the picket fence at a 5° angle from vertical? At a 10° angle?



The Force Table

Equipment and Setup: Force table with strings and 4 hangers, Set of masses

In this experiment, we will add vectors two ways: With the analytical method, we will calculate the magnitude and direction angle of the sum of two vectors from the magnitude and directions of the vectors we are adding. The force table will provide the experimental method, where we vary hanging weights to find the magnitude and direction angle of the vector sum.

Vector Basics

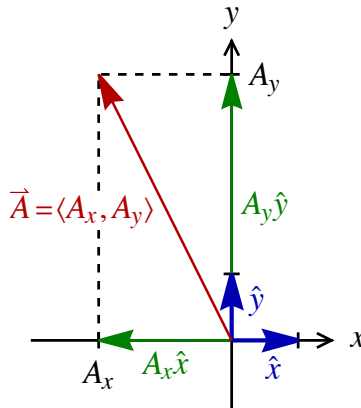
A vector is a physical quantity with both a magnitude and direction. A scalar has just a magnitude. Generally, the magnitude of a vector is a scalar that must be non-negative. A vector can be written in terms of components and the basis unit vectors, or in terms of just components using the angled bracket notation.

$$\vec{A} = A_x \hat{x} + A_y \hat{y} = A_x \hat{i} + A_y \hat{j} = \langle A_x, A_y \rangle$$

The x - and y -components of \vec{A} are A_x and A_y . In the different notations, the basis unit vectors in the x - and y -directions are

$$\hat{x} = \hat{i} = \langle 1, 0 \rangle \text{ and } \hat{y} = \hat{j} = \langle 0, 1 \rangle$$

respectively.



The magnitude of this is A . In two dimensions we can specify a direction by an angle θ , which we will refer to as the direction angle. This angle is unique, modulo 360° ; this means that if two angles differ by a multiple of 360° , they describe the same direction. We use the polar coordinates convention for angles, where an angle is measured counterclockwise from the positive x -axis. The magnitude and direction angle, A and θ , are related to the components, A_x and A_y , just as the polar coordinates, r and θ , are related to the rectangular coordinates, x and y . The components can be found in terms of the magnitude and direction angle:

$$A \text{ and } \theta \Rightarrow A_x \text{ and } A_y$$

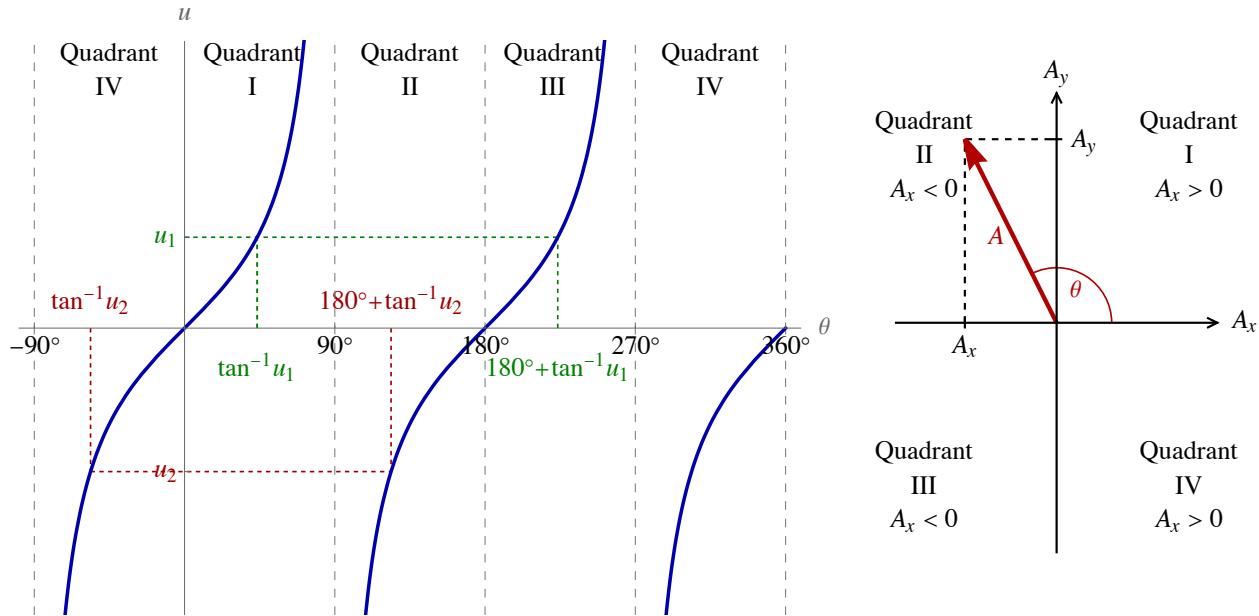
$$A_x = A \cos \theta \quad \text{and} \quad A_y = A \sin \theta \quad (\text{Equations 1})$$

and the magnitude and direction angle can be found in terms of the components:

$$A_x \text{ and } A_y \Rightarrow A \text{ and } \theta$$

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \begin{cases} \tan^{-1} \frac{A_y}{A_x} & \text{when } A_x > 0 \\ 180^\circ + \tan^{-1} \frac{A_y}{A_x} & \text{when } A_x < 0 \end{cases} \quad (\text{Equations 2})$$

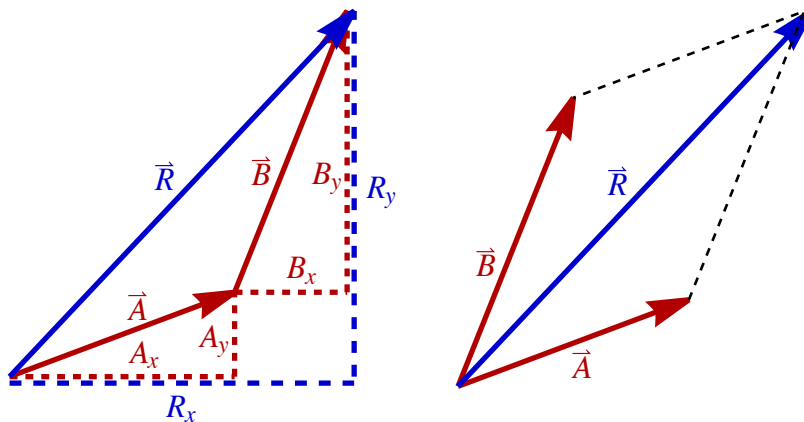
The subtlety in finding the direction angle follows from there being two angles satisfying $\tan \theta = A_y/A_x$, the arctangent of A_y/A_x and 180° added to that arctangent. The arctangent always gives an answer in quadrants I and IV. If the angle is in those quadrants, you use the arctangent. However, if the angle is in quadrants II or III, then you need to take the angle opposite; that is when you add 180° . The sign of A_x determines which is needed. When $A_x > 0$, the vector is in the first or fourth quadrants and when in the second or third quadrants, $A_x < 0$. See the figures below.



When we add vectors, the components of the sum are the sum of the components. If \vec{A} and \vec{B} are vectors then their sum, which we will call the resultant \vec{R} , is

$$\vec{A} = \langle A_x, A_y \rangle \text{ and } \vec{B} = \langle B_x, B_y \rangle \text{ then } \vec{R} = \vec{A} + \vec{B} = \langle A_x + B_x, A_y + B_y \rangle = \langle R_x, R_y \rangle$$

Graphically, we add \vec{A} and \vec{B} by putting the tail of \vec{B} at the tip of \vec{A} and the resultant \vec{R} is the vector from the tail of \vec{A} to the tip of \vec{B} . Alternatively, we can add the vectors using the parallelogram rule. Put the vector's tails together, complete the parallelogram and draw the vector to the opposite corner.



Forces and the Force Table

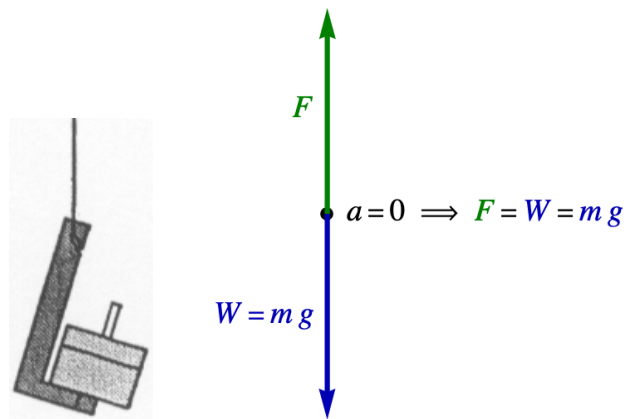
This experiment is about vectors, however the vectors we will add are force vectors. Because the experiment is designed to precede a discussion of forces in the lecture course, we must outline the aspects of

forces needed for the experiment. Since nothing will be moving in this experiment, we have what is known as static equilibrium; in static equilibrium all forces acting on a body must sum to zero: $\sum \vec{F} = \vec{0}$. Note that the zero-vector is the vector with zeros as components: $\vec{0} = \langle 0, 0 \rangle$.

We will hang masses on a string using a hanger with added masses. The total hanging mass is m , the sum of the added mass and the hanger's mass. Weight W is a force; it is the force of gravity acting on an object. In the SI system of units we measure force in newtons N and we measure mass in kilograms kg. The magnitude of the weight is related to the mass by:

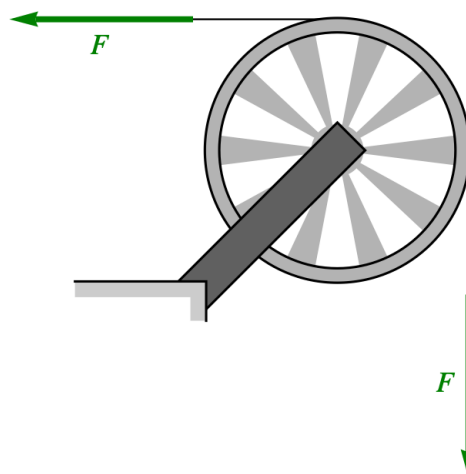
$$W = mg \text{ where } g = 9.80 \text{ m/s}^2$$

For the purposes of this experiment, we will avoid multiplying all masses by 9.80 m/s^2 to get the weight by using $g \cdot \text{kg}$ as our units of force. It follows that the hanging mass in kg corresponds to the same magnitude force in $g \cdot \text{kg}$. For instance, if we have a total hanging mass of 200 grams, that is 0.20 kg and thus corresponds to a weight of $0.20 g \cdot \text{kg}$.



Because there is no acceleration, the forces on the hanger must balance.

The other important fact about forces for this experiment is that for an ideal pulley, the string's tension F is the same on either side of the pulley.



An ideal pulley is frictionless and light; this means that the friction in the pulley is negligible, and the mass of the pulley is small compared to the other masses. (In this experiment, since there is no acceleration, the assumption that the pulley is light is not needed.)

Procedure for the Analytical Method

Vector Addition

For Vector Addition 1, 2 and 3, you will be given the magnitude and direction angle of the two (or three) force vectors you are summing. From these magnitudes and direction angles you find the vectors \vec{F}_1 and \vec{F}_2 . Then add these vectors to get the resultant $\vec{R} = \vec{F}_1 + \vec{F}_2$. You will then find the magnitude R and direction angle θ_R of the resultant vector, the sum of the force vectors.

An example will be helpful. Suppose you are given the magnitude and direction angles of \vec{F}_1 and \vec{F}_2 :

$$F_1 = 0.155 \text{ g}\cdot\text{kg}, \theta_1 = 25^\circ, F_2 = 0.275 \text{ g}\cdot\text{kg} \text{ and } \theta_2 = 135^\circ$$

Find the components of the force vectors using (Equations 1).

$$\vec{F}_1 = \langle F_{1x}, F_{1y} \rangle = \langle F_1 \cos \theta_1, F_1 \sin \theta_1 \rangle = \langle 0.14048, 0.06551 \rangle \text{ g}\cdot\text{kg}$$

$$\vec{F}_2 = \langle F_{2x}, F_{2y} \rangle = \langle F_2 \cos \theta_2, F_2 \sin \theta_2 \rangle = \langle -0.19445, 0.19445 \rangle \text{ g}\cdot\text{kg}$$

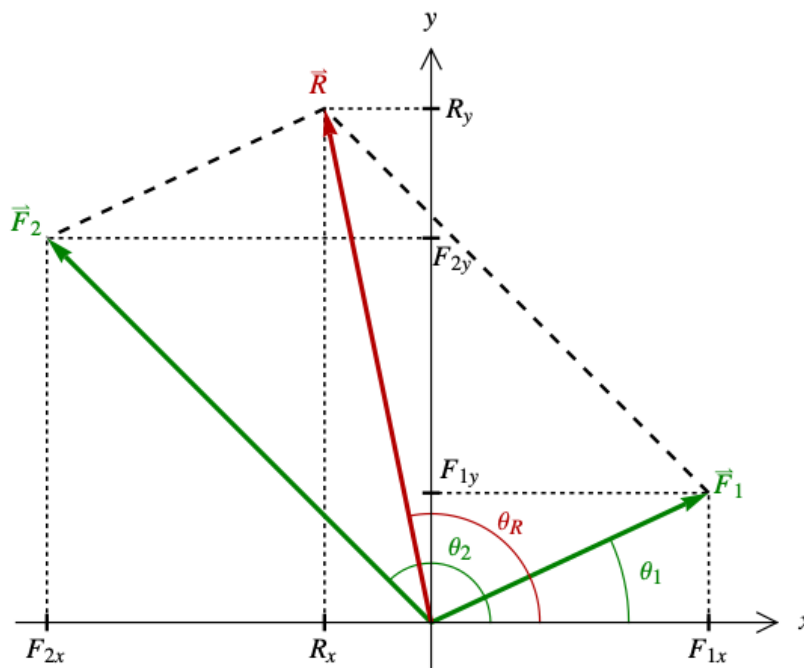
Adding these give the resultant \vec{R} and its components:

$$\vec{R} = \langle R_x, R_y \rangle = \vec{F}_1 + \vec{F}_2 = \langle -0.05398, 0.25996 \rangle \text{ g}\cdot\text{kg}$$

Using (Equations 2) we can find the magnitude and direction angle of the resultant.

$$R = \sqrt{R_x^2 + R_y^2} = 0.266 \text{ g}\cdot\text{kg} \text{ and } \theta_R = 180^\circ + \tan^{-1} \left(\frac{R_y}{R_x} \right) = 101.7^\circ$$

This is the result we are seeking. Note that the 180° was added to the arctangent because $R_x < 0$.



The vector diagram for the worked-out example

For Vector Addition 3, you will be adding three force vectors. Follow the procedure outlined above but now include the third force: $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$.

Vector Resolution

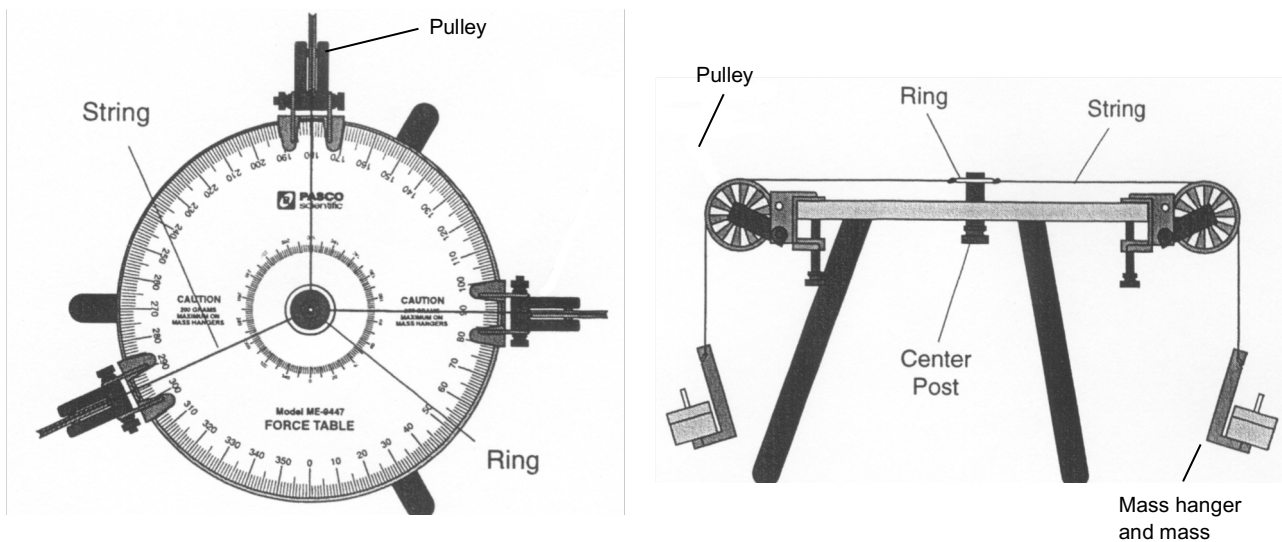
The last two parts of this experiment, Vector Resolution 1 and 2, involve finding the components of vectors. For each case you will be given F and θ , the magnitude and direction angle of some force vector, and you want to find the components F_x and F_y . Calculating these components is a simple application of (Equations 1):

$$F_x = F \cos \theta \quad \text{and} \quad F_y = F \sin \theta$$

Experimental Procedure - the Force Table

Setup

- Obtain a force table (with four pulleys and mass hangers) and a set of masses.
- Looking at the force table from the top, make sure that there is some visible gap between the pulleys and the edge of the force table. All pulleys should be able to rotate freely without rubbing against the edge of the force table. If the pulleys need adjustment, locate the horizontal screw near where the pulley meets edge of the force table. Loosen this screw and tilt the pulley outward until it no longer rubs against the edge of the table.
- Looking at the force table from the side, make sure that all four pulleys are at the same vertical height, so that all the strings lie in a single horizontal plane (or as nearly so as possible). If one or more pulleys need adjustment, use the screw referred to in the previous step to tilt the pulley(s) until all pulleys lie very nearly in a single horizontal plane.
- Hold the ring so that it is centered on the post and look down at the force table from the top. Make sure that all strings pull radially away from the center of the post. From time to time, the strings can slip from side to side along the circumference of the ring so that they are no longer radial. This results in some error in the angle giving the direction of the tensions acting on the ring. If one or more of your strings have slipped off-center, slide them so that all strings pull radially away from the post.



Top View and Side View of Force Table

Vector Addition

To experimentally add two vector, we use the force table. We cannot find the resultant directly. We find the third vector, called the *equilibrant*, that cancels the other two and balances the ring in the force table.

$$\vec{0} = \vec{F}_1 + \vec{F}_2 + \vec{E}$$

It follows that the resultant can then be found from the equilibrant:

$$\vec{R} = -\vec{E}$$

To set \vec{F}_1 and \vec{F}_2 set the pulleys at angles θ_1 and θ_2 . Then hang the weights F_1 and F_2 from the pulleys. If the force is $0.125 \text{ g}\cdot\text{kg}$, then you will hang total mass of $0.125 \text{ kg} = 125 \text{ grams}$. With those two strings pulling on the ring at the center, take a third string and with your hand find the direction needed to balance the other two forces. This is the direction angle of the equilibrant \vec{E} . Set the third pulley at that angle and vary the weights hanging from it until that force balances the other two. The magnitude of the hanging weight in $\text{g}\cdot\text{kg}$ is the magnitude of the equilibrant, E . But since $\vec{R} = -\vec{E}$ the magnitudes of \vec{R} and \vec{E} are equal, $R = E$. The vectors are in opposite directions, so the direction angle for the resultant θ_R is related to the direction angle of the equilibrant θ_E by $\theta_R = \theta_E \pm 180^\circ$. The magnitude of the resultant is the weight hanging. If the equilibrant is at 250° then the angle of the resultant is 70° .

There are four pulleys and strings but you will only need three for Vector Addition 1 and Vector Addition 2. In those cases, make sure the fourth string is not hanging with any tension; it is best to set that fourth hanger on the force table. Vector Addition 3 will involve adding three force vectors and that will use all pulleys and hangers.

To describe the experimental determination of R and θ using the force table, we will consider the same example given in the analytical method:

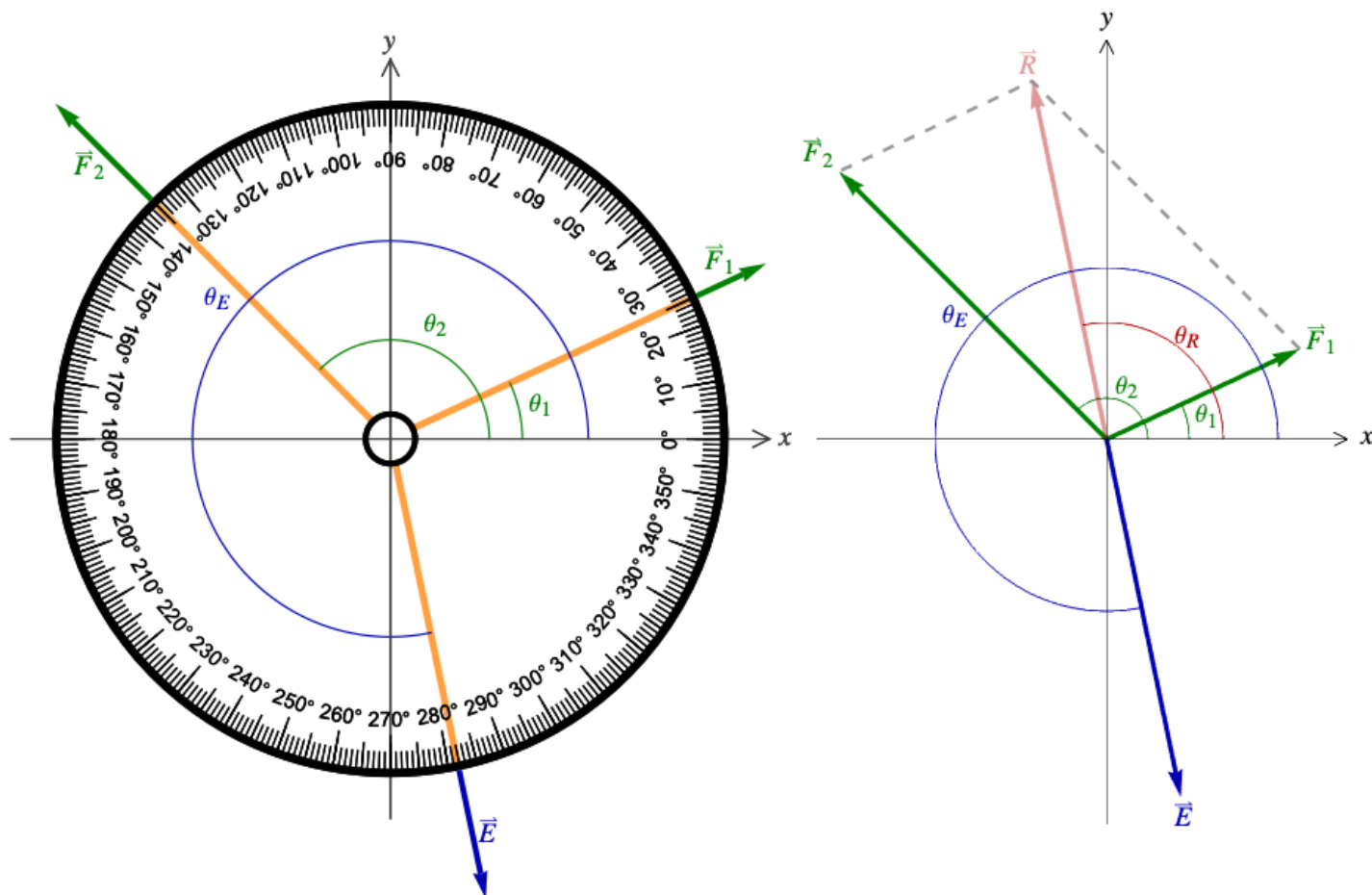
$$F_1 = 0.155 \text{ g}\cdot\text{kg}, \theta_1 = 25^\circ, F_2 = 0.275 \text{ g}\cdot\text{kg} \text{ and } \theta_2 = 135^\circ$$

Over pulleys set at 25° and 135° , hang $0.155 \text{ kg} = 155 \text{ grams}$ and $0.275 \text{ kg} = 275 \text{ grams}$, respectively. Take a third string and pull in varying directions until you find the direction needed to balance the forces from the other two strings. Suppose this is at 283° ; that then is the direction angle for the equilibrant. Set the third pulley there and start hanging masses over that third pulley until you get it to balance. The total mass hanging gives the magnitude of the equilibrant. Suppose this is $265 \text{ grams} = 0.265 \text{ kg}$; this then gives the magnitude of \vec{E} , the equilibrant, in $\text{g}\cdot\text{kg}$:

$$\theta_E = 283^\circ \text{ and } E = 0.265 \text{ g}\cdot\text{kg}$$

Since $\vec{R} = -\vec{E}$ we know that their magnitudes are equal and directions are opposite. This would give the values to record:

$$\theta_R = \theta_E - 180^\circ = 103^\circ \text{ and } R = E = 0.265 \text{ g}\cdot\text{kg}$$



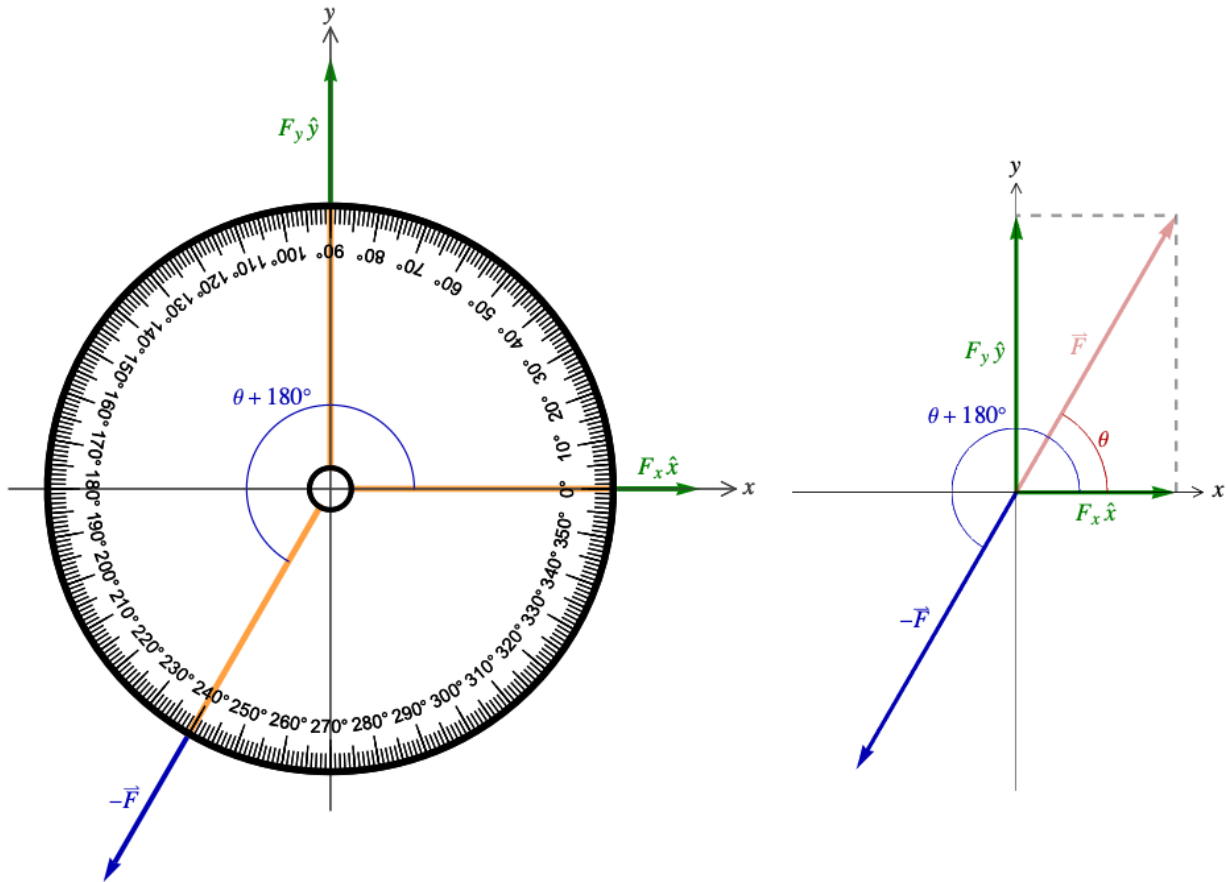
The diagram for the sample example

Vector Resolution

The last two parts of the experiment involve vector resolution, finding the components of a vector using the force table. With Vector Resolution 1, we have an acute angle which makes the procedure simple. To relate this to the Vector Addition part, think of the vector \vec{F} , with $F = 0.125 \text{ g}\cdot\text{kg}$ and $\theta = 60^\circ$, as the resultant vector \vec{R} ; it is the sum of vectors in the x - and y -directions:

$$\vec{F} = F_x \hat{x} + F_y \hat{y}$$

Set three pulleys at 0° , 90° and 240° . We then hang nothing at 60° but hang $0.125 \text{ kg} = 125 \text{ grams}$ opposite to that at 240° . This is $-\vec{F}$ which you should view as the equilibrant. But now we vary the weights hanging at 0° and 90° . Since the angle is acute, both components are positive; what is hanging at 0° is the x -component and what is hanging at 90° is the y -component.



The diagram for Vector Resolution 1

When the angle is not acute we can have negative components. This is the case in Vector Resolution 2. Hang $0.155 \text{ kg} = 155 \text{ grams}$ opposite to $\theta = 125^\circ$, then set other two pulleys along the axes as needed to balance $-\vec{F}$. To get the x -component set the pulley at either 0° or 180° and for the y -component set the pulley at 90° or 270° . The hanging weights will give $|F_x|$ and $|F_y|$. Be careful to include the appropriate signs for F_x and F_y .

Data Table

	Forces	Resultant - \vec{R} (magnitude and direction angle)	
		Experimental	Analytical
Vector Addition 1	$F_1 = 0.125 \text{ g}\cdot\text{kg}$ and $\theta_1 = 30^\circ$ $F_2 = 0.125 \text{ g}\cdot\text{kg}$ and $\theta_2 = 120^\circ$	$R =$ $\theta_R =$	$R =$ $\theta_R =$
Vector Addition 2	$F_1 = 0.075 \text{ g}\cdot\text{kg}$ and $\theta_1 = 20^\circ$ $F_2 = 0.125 \text{ g}\cdot\text{kg}$ and $\theta_2 = 120^\circ$	$R =$ $\theta_R =$	$R =$ $\theta_R =$
Vector Addition 3	$F_1 = 0.125 \text{ g}\cdot\text{kg}$ and $\theta_1 = 30^\circ$ $F_2 = 0.175 \text{ g}\cdot\text{kg}$ and $\theta_2 = 80^\circ$ $F_3 = 0.165 \text{ g}\cdot\text{kg}$ and $\theta_2 = 150^\circ$	$R =$ $\theta_R =$	$R =$ $\theta_R =$
Vector Resolution 1	$F = 0.125 \text{ g}\cdot\text{kg}$ and $\theta = 60^\circ$	$F_x =$ $F_y =$	$F_x =$ $F_y =$
Vector Resolution 2	$F = 0.155 \text{ g}\cdot\text{kg}$ and $\theta = 125^\circ$	$F_x =$ $F_y =$	$F_x =$ $F_y =$

Show your work. Use the back of this sheet or a separate sheet.

Questions

Question 1 Two vectors \vec{A} and \vec{B} have magnitudes A and B , respectively. What are the largest and smallest values of the magnitude of $\vec{A} + \vec{B}$, the sum of the two vectors? Give your answer in terms of A and B . Remember that the magnitude of a vector must be positive.

Question 2 Give a different derivation of the analytical results of Vector Addition 1, using the parallelogram rule and simple geometry. Note that since the two vectors have the same magnitude and are perpendicular, the parallelogram is a square.

Question 3 Using the values given in Vector Addition 2, find the magnitude and direction angle of the difference of the two vectors, $\vec{F}_2 - \vec{F}_1$.

Question 4 When experimentally determining the components of \vec{F} in Vector Resolution 2, at what angles were the pulleys placed? Give all three angles used.

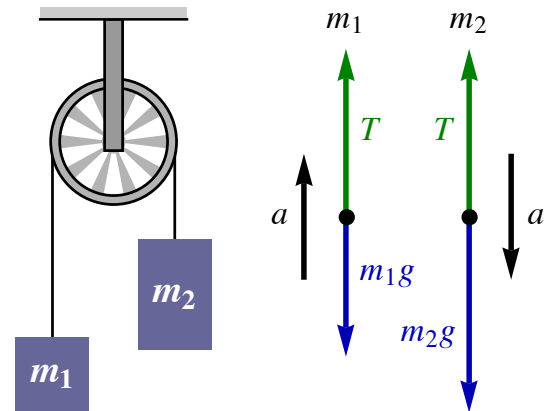
Atwood's Machine Experiment

Equipment and Setup: Smart Pulley, Smart Pulley clamp, Mass Hangers (2), Set of masses, String, Capstone file – Atwood's Machine.cap

Theory

Newton's second law $\vec{F}_{\text{net}} = m\vec{a}$ is the basis for mechanics; it relates the vector sum of all the forces acting on a body to the mass of the body and its acceleration vector. To apply the second law to any problem, the first step is to draw a free-body diagram for each mass; these are vector diagrams showing all the forces acting on a body. When forces are acting in one dimension only, we may write the expression without the vector arrows, with the caveat that one-dimensional vectors are real numbers with the sign giving the direction.

A standard example of applying the second law is Atwood's machine; this consists of two masses m_1 and m_2 connected by a light non-elastic string hanging over an ideal pulley. An ideal pulley is frictionless and light; this means that the rotational friction of the pulley can be neglected and the mass of the pulley, meaning the part of the pulley that rotates, is negligible compared to all the other masses in the problem. To the right is a drawing of the Atwood's machine set-up combined with free-body diagram for each mass. For this experiment we will take m_2 to be the larger of the two masses. This implies that m_2 will accelerate downward and m_1 will accelerate upward. Also, since the string is not elastic and cannot stretch, the magnitudes of both accelerations are the same. We will choose the direction of motion for the system to be positive, so downward is positive for m_2 and upward is positive for m_1 . Acting on both masses there is the tension in the string T acting upward and its weight m_1g or m_2g acting downward. The second law then gives the equations describing the motion of both masses.



$$F_{\text{net},1} = m_1 a \Rightarrow T - m_1 g = m_1 a \quad \text{and} \quad F_{\text{net},2} = m_2 a \Rightarrow m_2 g - T = m_2 a$$

Adding these two expressions together eliminates the tension T and allows us to solve for the acceleration a .

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

This will be referred to as the theoretical acceleration a_{theo} . For this experiment, we will label the numerator of this expression as the net force F_{net} and the denominator as the total mass m_{tot} .

$$F_{\text{net}} = (m_2 - m_1)g \quad \text{and} \quad m_{\text{tot}} = m_1 + m_2 \Rightarrow a_{\text{theo}} = \frac{F_{\text{net}}}{m_{\text{tot}}}$$

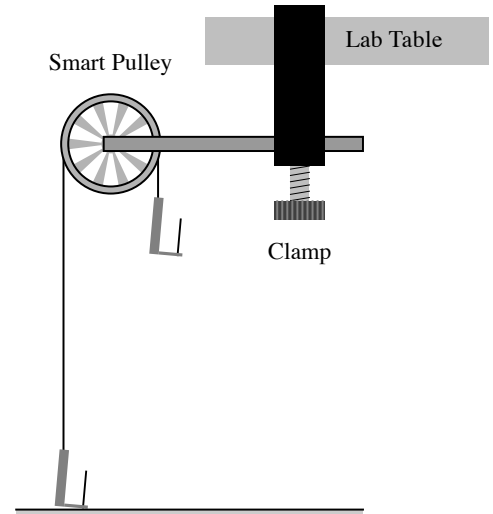
This experiment uses a “smart pulley” connected to the Pasco interface to measure the acceleration of the system. The smart pulley is a light low-friction pulley; what makes it “smart” is that it has a photogate. Just as the black stripes of the picket fence blocked the beam of the photogate in the Free Fall experiment, the pulley's spokes block the photogate's beam here. The calculation of the acceleration is the same as in the Free Fall experiment, except here Δx is the distance the string moves when the pulley rotates between the leading edges of the spokes. Since in the earlier experiment you completed the character-building exercise of calculating the velocity-time values from the raw data, we do not need to repeat that here. The Capstone programs knows the appropriate value of Δx for the smart pulley and magically gives the graph of velocity versus time, as in the Free Fall lab; but here it is not magic because we understand the calculation.

There are two parts of this experiment: In part A we will vary F_{net} while keeping m_{tot} fixed. Part B will vary m_{tot} keeping F_{net} fixed. Each run will use a different pair of masses m_1 and m_2 ; the Capstone program will give the velocity versus time graph and give the equation of best-fit line; this will tell you the acceleration. This acceleration will be called the experimental acceleration a_{exp} .

Procedure

Setup

1. Set up the equipment as shown to the right. Connect two hangers to a string that passes over the pulley as shown. Choose the length of the string so that one hanger is touching the floor when the other is just below the pulley.
2. Turn on the Pasco interface at your lab station and plug the photogate into Digital Channel 1.
3. In the “Physics Folder” on your computer desktop open Atwood’s Machine.cap .

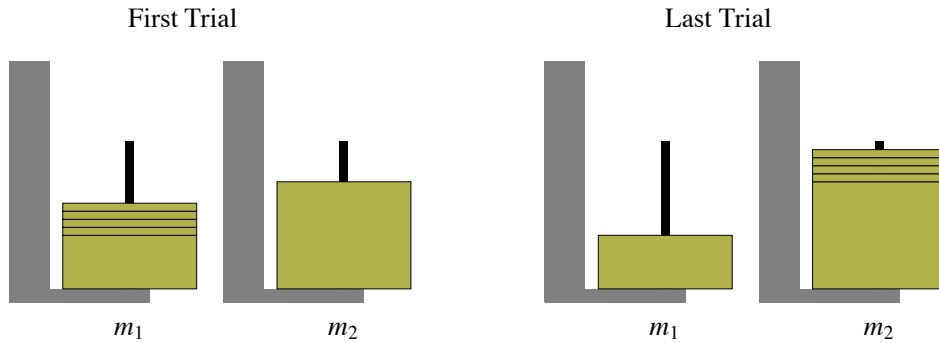


Data Recording – Finding Acceleration for both Parts A and B

1. The procedure for using the Capstone program is the same as in the Free Fall experiment. Here hold m_2 just below the pulley and m_1 just touching the floor. In this experiment m_2 *must always* be larger than m_1 .
2. It is essential while choosing the values of the masses to make sure that the acceleration is not too large or too small. The acceleration is too large when m_2 is much larger than m_1 . The problem with this situation is it will cause the string to slip on the pulley and the smart pulley then misreads the acceleration. If the acceleration is very small, then whatever friction there is in the pulley will become too significant. As a rule of thumb, try to keep the acceleration values between 0.5 m/s^2 and 5 m/s^2 . If one of the trials in parts A or B is outside of this range, it is fine, but if several are, the quality of your results will suffer.
3. When recording the masses you *must* include the mass of the hangers. The hanger masses contribute to m_{tot} and thus do not cancel. Measure the mass of the hangers.
4. In *Capstone*, click **Record** and release m_2 . When m_2 hits the floor, click the **Stop** button.
5. In *Capstone*, maximize the graph by clicking the “Scale to fit” tool above the graph (on left).
6. Click the “Highlighter” tool on top of the graph and drag the edges of the highlight box to select the “good” data. This is the linear part of the graph with a positive slope. The rest of the graph is what happened after m_2 hit the ground.
7. Fit the “good” data by clicking the “Curve Fits” drop down menu and selecting the *linear* fit. Record the slope in as a_{exp} in the appropriate table, either Table A or Table B.

(A) m_{tot} is fixed while F_{net} is varied.

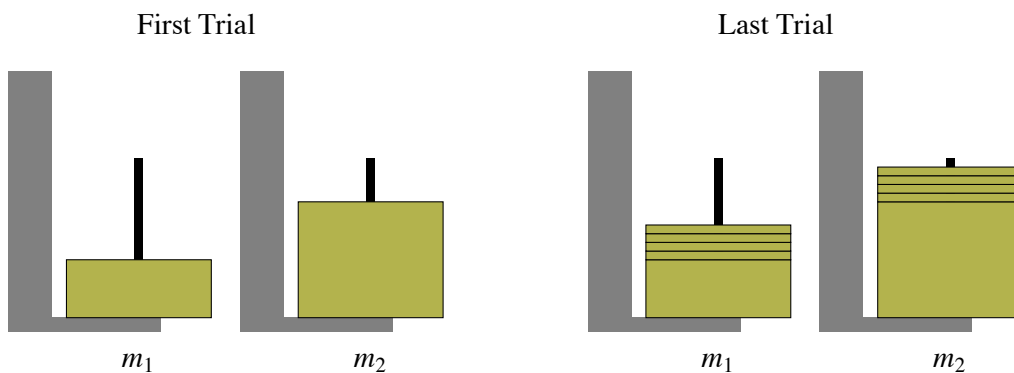
- To keep the total mass fixed while varying the net force, move small masses from one to the other. Start with m_1 with a larger mass and four smaller masses on top of it. Begin with m_2 a little larger than the initial m_1 . Between each trial move a small mass from m_1 to m_2 . The accelerations will increase in this process, so you want to begin with a small acceleration.



- Record your data in Table A and complete the table. Record the value of m_{tot} above the table. Complete the table by calculating and recording the net force, the theoretical acceleration and the percent error in the acceleration.
- Follow the directions below the table to graph the data.

(B) F_{net} is fixed while m_{tot} is varied.

- To keep the net force fixed while varying the total mass, add equal small masses to each mass between trials. Start with m_1 small but not tiny, and with m_2 large compared to m_1 . The accelerations will decrease in this process, so you want to begin with a larger acceleration.



- Record your data in Table B and complete the table. Record the value of F_{net} above the table. Complete the table by calculating and recording the total mass, the theoretical acceleration, and the percent error in the acceleration. Also, to assist you in graphing, calculate $1/a_{\text{exp}}$.
- Follow the directions below the table to graph the data.

Atwood's Machine – Data Sheet

Name _____ Group _____

(A) Variation of Acceleration with F_{net} Keeping m_{tot} Fixed

$m_{\text{tot}} =$ _____ kg

Table A

m_1 (kg)	m_2 (kg)	a_{exp} (m/s ²)	F_{net} (N)	a_{theo} (m/s ²)	% error in a

Graph F_{net} versus a_{exp} and include the best-fit line. Find the slope of this line and compare the slope with m_{tot} . Give the % error.

(B) Variation of Acceleration with m_{tot} Keeping F_{net} fixed

$F_{\text{net}} = \underline{\hspace{2cm}}$ N

Table B

m_1 (kg)	m_2 (kg)	a_{exp} (m/s ²)	m_{tot} (kg)	a_{theo} (m/s ²)	% error in a	$1/a_{\text{exp}}$ (s ² /m)

Graph m_{tot} versus $1/a_{\text{exp}}$ with the best-fit line. Find the slope of this line and compare the slope with F_{net} . Give the % error.

Numerical and Symbolic Methods in One Dimensional Dynamics

Newton's second law is the basis for dynamics. It is the rule for evolving a dynamical system forward in time. Here we will discuss the problem of one dimensional motion. The second law in one dimension states:

$$F(x, v, t) = m a,$$

where F is the net force which is a function of x , the position, v , the velocity, and perhaps t , the time. If at some instant t the position and velocity are given by x and v , then an infinitesimal time step dt later the values change by the rule:

$$\begin{aligned}t &\rightarrow t + dt \\x &\rightarrow x + v dt \\v &\rightarrow v + \frac{F(x,v,t)}{m} dt.\end{aligned}$$

In a numerical simulation we approximate the infinitesimal time step with a small time step Δt . We will label an instant by i and the rules for taking a time step are

$$\begin{aligned}t_i &\rightarrow t_{i+1} = t_i + \Delta t \\x_i &\rightarrow x_{i+1} = x_i + v_i \Delta t \\v_i &\rightarrow v_{i+1} = v_i + \frac{F(x_i, v_i, t_i)}{m} \Delta t.\end{aligned}$$

We may numerically evolve a problem by starting with the initial conditions: t_0 , x_0 and v_0 , and by giving the mass m and the force function F .

The second law is the rule that gives the differential equation for the motion. A differential equation is an equation for a *function* that involves the function and its derivatives. After the third semester of calculus students take a course in differential equations to learn how to solve them. Here we will just use *Mathematica* to solve them.

The second law gives a second order differential equation for the position as a function of time $x(t)$:

$$\frac{d^2}{dt^2} x(t) = \frac{1}{m} F(x(t), \frac{d}{dt} x(t), t)$$

Note that the order of a differential equation is the highest number of derivatives of the unknown function.

(A) Free Fall

For free fall we will take down as the positive direction and the force becomes

$$F(x, v, t) = m g.$$

We will take the initial conditions to be

$$t_0 = 0, x_0 = 0 \text{ and } v_0 = 0.$$

We will also take the mass to be 2 kg. We will choose the final time to be $t = 2$ s and choose our time interval at first to be the relatively large value of $\Delta t = 0.2$ s.

To evaluate *Mathematica* input place the cursor anywhere in an input cell and **hit Shift+Enter**. This will be done for all *Mathematica* input that follows.

```
t[0] = 0; x[0] = 0; v[0] = 0;
t[final] = 2; dt = 0.2;
m = 2; g = 9.8;
F[x_, v_, t_] := m g
```

The following is the *Mathematica* code for finding the numerical solution. XNum and VelNum are the functions that interpolate the discrete position and velocity points with continuous functions.

```

iMaximum = (t[final] - t[0]) / Δt;
Do[
  t[i + 1] = t[i] + Δt;
  x[i + 1] = x[i] + v[i] Δt;
  v[i + 1] = v[i] + Δt F[x[i], v[i], t[i]] / m;
  {i, 0, iMaximum - 1}
];
XNum = Interpolation[Table[{t[i], x[i]}, {i, 0, iMaximum}]];
VelNum = Interpolation[Table[{t[i], v[i]}, {i, 0, iMaximum}]];

```

The following is the *Mathematica* code for finding the symbolic solution. The function XSym is the result of DSolve, the *Mathematica* function for solving a differential equation. VelSym is the symbolic velocity function, found by differentiating the position XSym.

```

XSym = DSolve[{F[X[t], X'[t], t] == m X''[t], X[0] == x[0], X'[0] == v[0]}, X, t][[1, 1, 2]];
VelSym = XSym';
Print["x(t) = ", XSym[t]]
Print["v(t) = ", VelSym[t]]
x(t) = 4.9 t2
v(t) = 9.8 t

```

This is the code for plotting the numerical solution, the wide yellow line, and the symbolic solution as a narrow blue line. The first plot is x vs. t and the second is v vs. t .

```

GraphicsColumn[{Plot[{XNum[T], XSym[T]}, {T, 0, t[final]}, PlotStyle →
  {{RGBColor[1, 1, 0.4`], Thickness[0.02`]}, {RGBColor[0, 0, 0.6`], Thickness[0.008`]}},
  AxesLabel → {"t", "x"}, Background → GrayLevel[0.6`]],
  Plot[{VelNum[T], VelSym[T]}, {T, 0, t[final]}, PlotStyle →
  {{RGBColor[1, 1, 0.4`], Thickness[0.02`]}, {RGBColor[0, 0, 0.6`], Thickness[0.008`]}},
  AxesLabel → {"t", "v"}, Background → GrayLevel[0.6`]]]

```

The *Mathematica* function CalculateAndGraph is now defined to follow the above procedure of calculating the numerical and symbolic solutions, and then graphing those solutions.

```

CalculateAndGraph := (iMaximum =  $\frac{t[\text{final}] - t[0]}{\Delta t}$ ;
Do[t[i + 1] = t[i] + Δt;
  x[i + 1] = x[i] + v[i] Δt;
  v[i + 1] = v[i] +  $\frac{\Delta t F[x[i], v[i], t[i]]}{m}$ ; , {i, 0, iMaximum - 1}];
XNum = Interpolation[Table[{t[i], x[i]}, {i, 0, iMaximum}]];
VelNum = Interpolation[Table[{t[i], v[i]}, {i, 0, iMaximum}]];
XSym =
  Quiet[DSolve[{F[X[t], X'[t], t] == m X''[t], X[0] == x[0], X'[0] == v[0]}, X, t][[1, 1, 2]];
VelSym = XSym';
Print["x(t) = ", XSym[t]];
Print["v(t) = ", VelSym[t]];
GraphicsColumn[{Plot[{XNum[T], XSym[T]},
  {T, 0, t[final]}, PlotStyle → {{RGBColor[1, 1, 0.4`], Thickness[0.02`]},
  {RGBColor[0, 0, 0.6`], Thickness[0.008`]}}, AxesLabel → {"t", "x"},
  Background → GrayLevel[0.6`]], Plot[{VelNum[T], VelSym[T]}, {T, 0, t[final]},
  PlotStyle → {{RGBColor[1, 1, 0.4`], Thickness[0.02`]}, {RGBColor[0, 0, 0.6`],
  Thickness[0.008`]}}, AxesLabel → {"t", "v"}, Background → GrayLevel[0.6`]]}]

```

Note that with the very crude value of $\Delta t = 0.2$ s, where there are only 10 intervals, the symbolic and numerical values are quite different. To improve this we must choose a smaller Δt to make a more accurate numerical approximation. Now we will try $\Delta t = 0.02$ s and redo the numerical solutions and plots.

```

Δt = 0.02;
CalculateAndGraph

```

(B) Motion with Viscous Damping

Viscous damping is a resistive force R that is proportional to the velocity $R = -b v$. Take the only force to be this. The force function becomes:

$$F(x, v, t) = -b v.$$

Below is a repeat of the *Mathematica* code for free fall. Modify it to change the force function to this and change the initial conditions and other parameters to

$$t_0 = 0, x_0 = 0, v_0 = 3 \frac{\text{m}}{\text{s}}, m = 2 \text{ kg and } b = 5 \frac{\text{kg}}{\text{s}}.$$

```

t[0] = 0; x[0] = 0; v[0] = 0;
t[final] = 2; Δt = 0.02;
m = 2; g = 9.8;
F[x_, v_, t_] := m g
CalculateAndGraph

```

(C) Free Fall with Viscous Damping

Now combine free fall with viscous damping. The force function becomes:

$$F(x, v, t) = m g - b v.$$

Once again, below is a repeat of the *Mathematica* code for free fall. Modify it to change the force function to this and change the initial conditions and other parameters to

$$t_0 = 0, x_0 = 0, v_0 = 0, m = 2 \text{ kg}, g = 9.80 \frac{\text{m}}{\text{s}^2} \text{ and } b = 5 \frac{\text{kg}}{\text{s}}.$$

```
t[0] = 0; x[0] = 0; v[0] = 0;
t[final] = 2; Δt = 0.02;
m = 2; g = 9.8;
F[x_, v_, t_] := m g
CalculateAndGraph
```

(D) Free Fall with Quadratic Damping

For free fall at sufficiently small speeds viscous damping is an accurate model. As speeds increase and the flow of air becomes turbulent, the resistive force becomes quadratic. $R = -c v^2$. Combining quadratic friction with free fall we get:

$$F(x, v, t) = m g - c v^2.$$

Once again, below is a repeat of the *Mathematica* code for free fall. Modify it to change the force function to this and change the initial conditions and other parameters to

$$t_0 = 0, x_0 = 0, v_0 = 0, m = 2 \text{ kg}, g = 9.80 \frac{\text{m}}{\text{s}^2} \text{ and } c = 0.5 \frac{\text{kg}}{\text{m}}.$$

```
t[0] = 0; x[0] = 0; v[0] = 0;
t[final] = 2; Δt = 0.02;
m = 2; g = 9.8;
F[x_, v_, t_] := m g
CalculateAndGraph
```

(E) Oscillations with a Spring and Mass

A spring produces a force that is proportional to the amount the spring is stretched. If it is stretched from its equilibrium position by x then $F \propto x$. We define k , the spring constant, as the constant of proportionality and write: $F = -k x$. The minus sign indicates that this is the force *of* the spring on the mass and not the force *on* the spring.

$$F(x, v, t) = -k x.$$

Once again, below is a repeat of the *Mathematica* code for free fall. Modify it to change the force function to this and change the initial conditions and other parameters to

$$t_0 = 0, x_0 = 2, v_0 = 0, m = 2 \text{ kg and } k = 90 \frac{\text{N}}{\text{m}}.$$

```
t[0] = 0; x[0] = 0; v[0] = 0;
t[final] = 2; Δt = 0.02;
m = 2; g = 9.8;
F[x_, v_, t_] := m g
CalculateAndGraph
```

If the error is too large between the symbolic solution, the blue line, and the numerical solution, the yellow line then you will need a smaller value of Δt . Try $\Delta t = .002$. To repaste the above input move the mouse cursor below this text and type *Ctrl-L*.

(F) Damped Oscillations with a Spring and Mass

Now combine the spring with viscous damping. Here you should see oscillations that damp out with time.

$$F(x, v, t) = -kx - bv.$$

Once again, below is a repeat of the *Mathematica* code for free fall. Modify it to change the force function to this and change the initial conditions and other parameters to

$$t_0 = 0, x_0 = 2, v_0 = 0, m = 2 \text{ kg}, k = 60 \frac{\text{N}}{\text{m}} \text{ and } b = 5 \frac{\text{kg}}{\text{s}}.$$

Modify the Δt as in the previous part.

```
t[0] = 0; x[0] = 0; v[0] = 0;  
t[final] = 2; Δt = 0.02;  
m = 2; g = 9.8;  
F[x_, v_, t_] := m g  
CalculateAndGraph
```

Numeric and Symbolic Methods in 1-D Dynamics

Open this file on the computer: Physics Folder → Mathematica → Dynamics in 1D.nb

Follow the instructions in the file and you will eventually generate a graph of $v(t)$ and $x(t)$ for each part (A) → (F). You do not need to print these out, but show them to your instructor and have them verify that you have done them correctly. Instructor Initials _____

In addition to generating the graphs in Mathematica, there are additional questions below for you to work out.

(B) Motion with Viscous Damping

Differential Equation: $\sum F = -bv = m \frac{dv}{dt}$

Guess: $v(t) = Ae^{-\beta t}$

Initial Conditions: $v(0) = v_0$

Determine the unknown constants A and β in terms of the known constants m , b , and v_0 , then give the complete expression for time-dependent velocity.

(C) Free Fall with Viscous Damping

From the graphs in part (C), numerically estimate the terminal velocity. In terms of the values of the parameters given in part (C), calculate the terminal velocity and compare with your estimate.

(D) Free Fall with Quadratic Damping

From the graphs in part (D), numerically estimate the terminal velocity. In terms of the values of the parameters given in part (D), calculate the terminal velocity and compare with your estimate.

(E) Oscillations with a Spring and Mass

Differential Equation: $\sum F = -kx = m \frac{d^2x}{dt^2}$

Guess: $x(t) = A \cos(\omega t) + B \sin(\omega t)$

Initial Conditions: $x(0) = x_0 \quad v(0) = 0$

Determine the unknown constants A , B , and ω in terms of the known constants m , k , and x_0 , then give the complete expression for time-dependent position.

Friction

Equipment and Setup: Friction Trays (3 types), 2 cart masses, Set of masses, Hanger and string, Table Clamp, rod and super pulley, Pasco interface, Capstone file – Friction.cap

In this experiment you will determine the coefficient of kinetic friction between a pair of surfaces. You will repeat this for several pairs of surfaces. You will also design an experiment to determine the static coefficient of friction between a pair of surfaces.

Theory

A mass, m_1 , on a horizontal surface is connected by a pulley to a hanging mass, m_2 . We assume the pulley has negligible mass, that the rope does not slip on the pulley, and that there is negligible friction between the pulley and its axis. The acceleration of the system can be computed from Newton's 2nd law

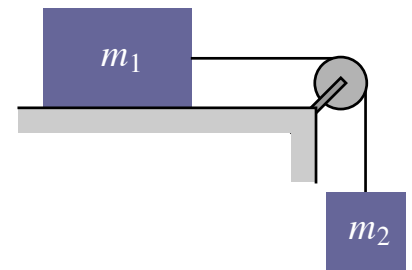
$$F_{net} = m_{sys}a$$

where $m_{sys} = m_1 + m_2$ is the mass of the system.

Assuming the system is in motion, this can be written as

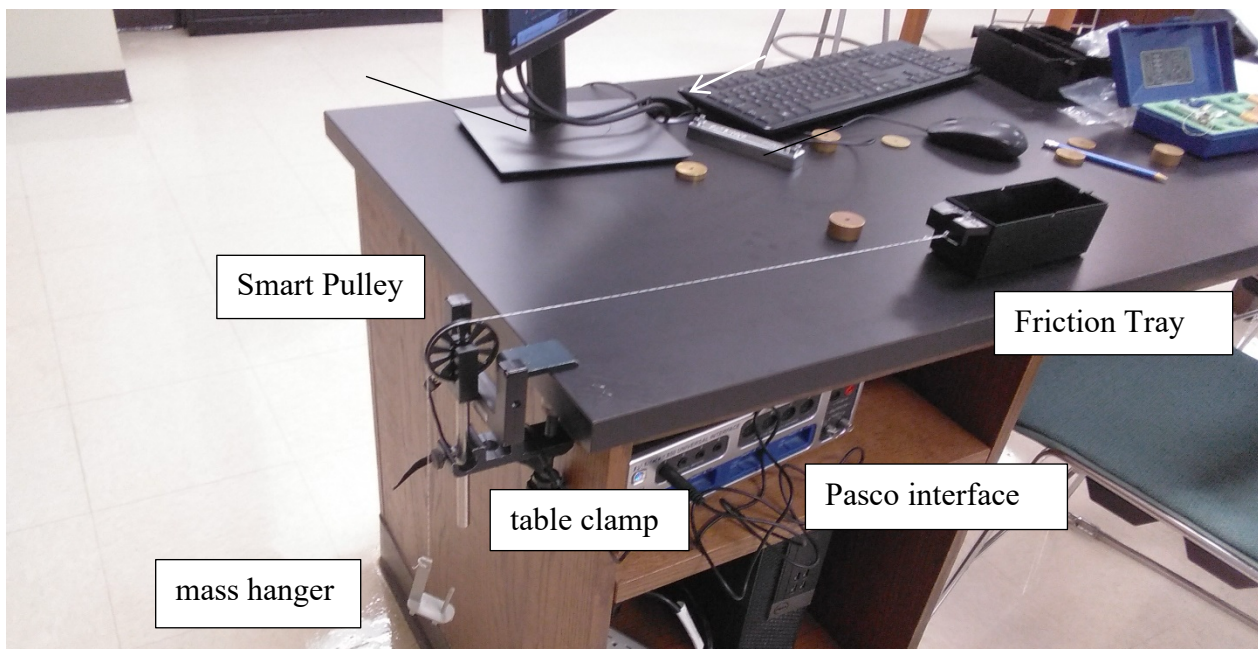
$$m_2g - f_k = (m_1 + m_2)a.$$

where $f_k = \mu_k m_1 g$ is the kinetic friction between m_1 and the horizontal surface.



Procedure

Setup



(S.1) Use a scale to find the mass of a plastic-bottom friction tray and one other friction tray—choose either felt or cork. Note: all three trays are made of black plastic, but the bottom of the plastic-bottom tray is white.

(S.2) Use the table clamp to attach the rod with super pulley to the end of the table. Plug the photogate into digital input 1 on the interface box. Turn on the interface box and start the *Capstone* file Friction.cap in Physics/Capstone folder. If you spin the pulley, the light on the back of the photogate should blink. The photogate detects when the spokes block an LED light shining into a detector. The diameter of the pulley is already in the computer, so it can use the angular speed of the pulley to determine linear speed of the string on the outer edge of the pulley, and thus, determine the speed of the system.

(S.3) Cut a string about 1 m long. Tie one end with an overhand loop and use the loop to attach the string to the plastic bottom friction tray. Attach the other end to a hanger by wrapping the string around the slot until it catches. Place the string over the pulley. Put one cart mass in the tray. Place a small mass on the hanger so that it hangs evenly, but the tray does not slide when released from rest. Adjust the pulley so that it is straight, and the string attached to the tray is horizontal.



overhand loop



connect friction tray



connect hanger

(A) Coefficient of Kinetic Friction: changing the normal force

(A.1) Place mass on the hanger and release the friction tray. **WARNING:** Block the table clamp with your hand so that the tray does not run into it! You can also put the second cart mass in front of the table clamp for added protection. If the friction tray does not move, add more mass to the hanger.

Once you have the system working:

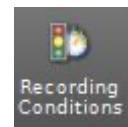
(A.2) Record the mass m_1 (tray + cart mass). Record mass m_2 (mass on hanger + 5g for hanger).

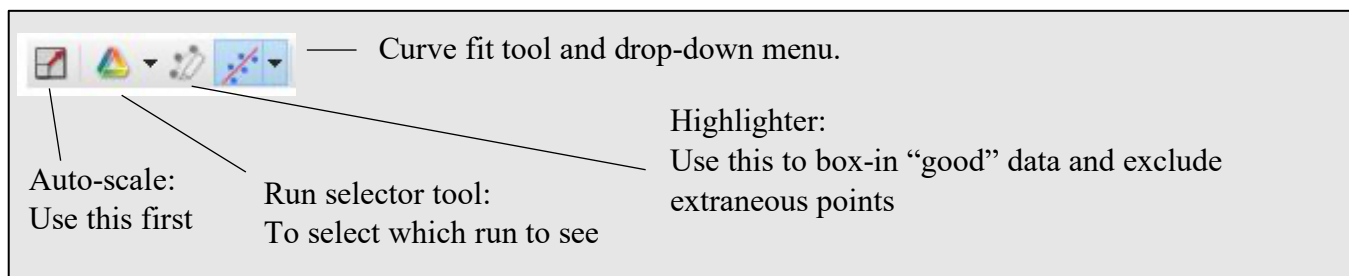
(A.3) Pull the friction tray back to about 50cm away from the pulley. Stop the hanger from swinging. Block the pulley with your hand so that the tray does not hit it. Click record and release the tray. Capstone should stop recording automatically.

Note: The start and stop conditions are set for 5cm and 30cm respectively. If you have trouble with them, you can adjust them or turn them off (the control is below the graph display.)

(A.4) Adjust the display so you can see the entire graph. Use the graph controls above the graph to select the "good" data. Fit the graph with a linear fit and record the slope as a_{exp} . If the graph is not linear (generally when $a_{exp} < 0.500 \text{ m/s}^2$) add a little mass to the hanger and try again.

Note: Controls for adjusting the graph are located above the graph to the left. You may need to click on the graph to see them. (See image below.)





To expand the graph: Mouse over either axis until you get a double arrow. Use mouse scroll or click and drag to expand or contract the axis.

To recenter the graph: Mouse over the graph until you get a hand. Click and drag to move the graph.

(A.5) Record data for 5 runs with different values of hanging mass, m_2 . Try to keep the acceleration below 3.00 m/s^2 so that the cart does go flying when it is stopped.

(A.6) Add the second cart mass to the friction tray and record data for 5 runs with different values of m_2 . Try to use the same values of hanging mass m_2 as previously. Although it may not be possible to use *all* the same values of m_2 , try to use at least 2 of the same values. Note: get all your data before you start analyzing.

(B) Coefficient of kinetic friction: changing surfaces

(B.1) Disconnect the hanging mass, detach the string from the friction tray. Connect the second friction tray to the system. Place one cart mass in the tray. Record m_1 (tray + added mass).

Note: The system works best if the mass in the tray is distributed evenly and does not slide—hence the use of the cart mass. However, on some lab tables, this may be too much weight, especially for the cork bottom tray. You may need to use a smaller mass from your mass set in this case.

(B.2) Repeat the experiment. Record data for 5 runs with different values of hanging mass m_2 .

(C) Coefficient of Static friction

Devise an experiment to determine the coefficient of static friction between two surfaces, one of the friction trays and the table. Make three different measurements of this coefficient, record data and determine an average value of this coefficient of static friction. Take notes. You will need to present a brief explanation of your experiment and present your data and results.

Friction – Data Sheet

Name _____ Group _____

Fill in the tables. Remember to include proper units at the top of columns.

Part A: Changing Normal Force

write the equation used to solve for f_k _____

write the equation used to solve for μ_k _____

Plastic base friction tray

Table A1. m_1 _____ kg

m_2 ()	a_{exp} ()	f_k () computed value	μ_k computed value
Average			

Table A2. m_1 _____ kg

m_2 ()	a_{exp} ()	f_k () computed value	μ_k computed value
Average			

Table B: Coefficient surface

Table B. Base of friction tray material (felt, cork) _____

m_1 _____ kg

m_2 ()	a_{exp} ()	f_k () computed value	μ_k computed value
		Average	

Data Part C: Create a table and record your data for part C in the space below. You will need this to answer question C-1 on the last page. Don't leave class without the data!

Questions

AB-1. What are the units of the coefficient of kinetic friction?

AB-2. How would the results of the experiment differ if the mass m_1 were initially moving in the opposite direction (away from the pulley)? Explain. Would you still be able to find the kinetic coefficient of friction in this case?

AB-3. Write the equation used to determine the *force* of kinetic friction. Should changing the hanging mass, m_2 change the *force* of kinetic friction on the tray? Do your experimental results support your assumption? (Be clear about HOW your results DO or DO NOT support the theory.)

AB-4. Write the equation used to determine the acceleration of the system. If you changed the normal force on a tray (m_1g) but did not change the hanging mass (m_2) should this change the force of friction on the tray? Should this change the value of coefficient of kinetic friction between the tray and the horizontal surface? Do your results from tables A1 and A2 support these claims? (Be clear about HOW your results DO or DO NOT support the theory.)

Data recording and Question for part C.

C-1. Briefly describe the procedure you used in part C to find the static coefficient of friction (one paragraph). From you data recorded in part C of the data recording section, derive the static coefficient for friction and present your results.

Systems and Friction

Equipment: Air track with air supply and hose, Glider, Accessory kit, Set of masses, Hanger and string, Table Clamp, rod and super pulley, Pasco interface, Capstone file – Forces and Friction.cap

In this experiment we will determine the coefficient of kinetic friction between the glider and the air track.

Theory: A mass, m_1 , on a horizontal surface is connected by a pulley to a hanging mass, m_2 . We assume the pulley has negligible mass, that the rope does not slip on the pulley, and that there is negligible friction between the pulley and its axis. The acceleration of the system can be computed from Newton's 2nd law

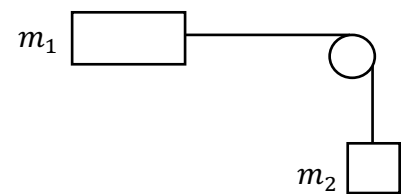
$$F_{net} = m_{sys}a$$

where $m_{sys} = m_1 + m_2$ is the mass of the system.

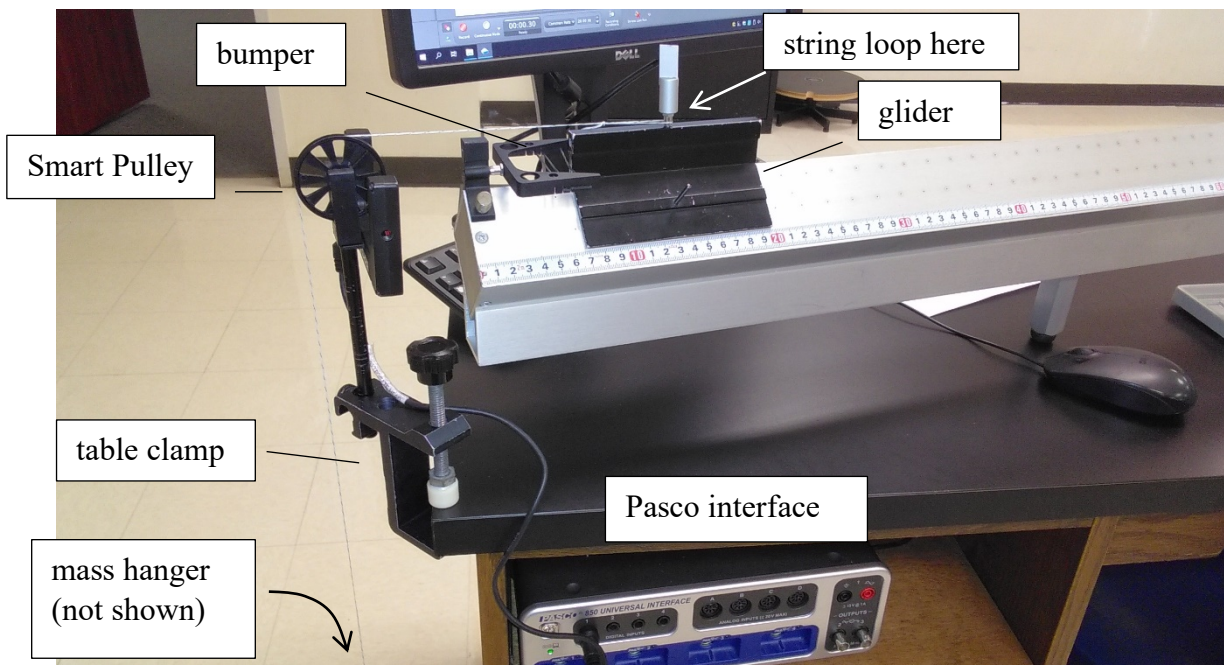
Assuming the system is in motion, this can be written as

$$m_2g - f_k = (m_1 + m_2)a.$$

where $f_k = \mu_k m_1 g$ is the kinetic friction between m_1 and the horizontal surface.



Setup:



1. Place the air track on the table and connect the air hose. The other end should be at the edge of the table closest to the interface box. Check that the section of track on this end is level. (Use the spirit level or, with air turned

on, glider stays at rest for a little while when placed at rest on track. It is not necessary for the *entire* track to be level.) Place any connector in the top slot of the glider. Use a scale to find the mass of the glider. You may wish to use a rubber-band bumper on the end of the track (lower hole).

2. Use the table clamp to attach the rod with super pulley to the end of the table. Cut a string about 1 m long. Tie a loop in one end, attach the other end to a hanger. Place the loop over the connector on the top of the glider and hang the string over the pulley. Adjust the setup until everything is aligned and the string is fairly level. You may need to angle the table clamp a little to get the screw out of the way of the hanging mass.

3. Plug the photogate into digital input 1 on the interface box. Turn on the interface box and start the *Capstone* file called `Friction.cap` in Physics/Capstone folder. If you spin the pulley, the light on the back of the photogate should blink. The photogate detects when the spokes block a light (not visible) shining into a detector. The diameter of the pulley is already in the computer, so it can convert the angular motion of the pulley into linear movement of the string on the outer edge of the pulley, and thus, the movement of the system.

(A) Accelerating without friction.

1. Turn the air supply on. Adjust it to a low setting. The glider should coast without slowing down when no string is attached.

2. Attach the string and put a small mass on the hanger, or leave it empty. Record the hanging mass, m_2 . REMEMBER to add 5g for the hanger.

3. Pull the glider back to about 50cm away from the end stop. Stop the hanger from swinging. Click record and release the glider. It should stop recording automatically.

Note: The start and stop conditions are set for 5cm and 30cm respectively. If you have trouble with them, you can adjust them or turn them off (the control is below the graph display.)

Note: If the glider gets moving too fast, you can stop it by hand.



4. Adjust the display so you can see the entire graph. Fit the graph with a linear fit and record the slope as a_{exp} .

Notes: Controls for adjusting graph are located above the graph to the left. You may need to click on the graph to see them.

Auto-scale: Use this first

Run selector tool: To select which run to see

Curve fit tool and drop-down menu.

Highlighter: (generally not needed)
If you have some extraneous data points, use this to box-in "good" data so you don't fit the entire run

Mouse over either axis until you get a double arrow. Use mouse scroll or click and drag to adjust axis.

Mouse over graph until you get a hand. Click and drag to move graph.

5. Record a total of 4 runs with different values of hanging mass, m_2 . Mass increments should be small, around 5g, so that the acceleration does not get too large.

(B) Coefficient of Kinetic Friction

1. Turn off the air supply. Move the glider back a bit and place mass on the hanger until the glider breaks free of static friction and starts to move. Try a trial run. If the graph does not look fairly linear, add more mass and try again.
2. Once the graph looks linear, record m_2 and a_{theo} . Increase m_2 . Record 4 runs with different values of hanging mass, m_2 . Note: you will need to make the mass increments much larger this time, around 20g.

Name _____ Group _____

Systems and Friction Datasheet

Fill in the tables. Remember to include proper units at the top of columns. Table A: There is no friction, so $f_k = 0$. Percent error = $\frac{theo-exp}{theo} \times 100\%$. Table B: Use a spreadsheet to find average and standard deviation. (Excel: use StDev or StDevS) Percent StDev = $\frac{standard\ deviation}{Average} \times 100\%$.

Table A: Accelerating without Friction

m_1 _____ kg

m_2 ()	a_{exp} ()	a_{theo} ()	percent error

Table B: Coefficient of kinetic friction

m_1 _____ kg

m_2 ()	a_{exp} ()	f_k () computed value	μ_k computed value
		Average	
		Standard Deviation	
		Percent StdDev	

Question 1 Explain how the results of Part (A) would be affected if the pulley has non-negligible mass. In your case, was it a valid to assume that the mass of the pulley was negligible?

Question 2 In Part (B), how would the results of the experiment differ if the mass m_1 were initially moving in the opposite direction (away from the pulley)? Could you still find the kinetic coefficient of friction in this case? Explain.

Question 3 How is a percent standard deviation more useful than a standard deviation in determining the validity of one's results?

Question 4 How could you use this equipment to find the static coefficient of friction between glider m_1 and the track? Briefly describe an experiment to accomplish this.

Energy and Systems

Equipment: Air track, air supply with hose, Glider, Accessory kit, Photogate on stand, Set of masses, Hanger and string, *Capstone* file – Energy.cap

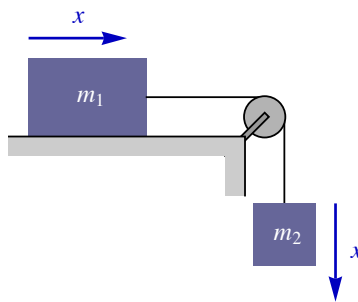
In this experiment we will analyze the conversion of potential energy into kinetic energy in a system of two masses.

Theory: A mass, m_1 , on a horizontal surface is connected by a pulley to a hanging mass, m_2 . We assume the pulley has negligible mass, that the rope does not slip on the pulley, and that there is negligible friction between the pulley and its axis. Let's start with the work and energy equation

$$W_{nc} = \Delta K + \Delta U$$

The first term, W_{nc} , is the work done by non-conservative forces. If kinetic friction is negligible, then this is zero. Although gravity does work to the system and the hanging mass, m_2 moves up or down, gravity is a conservative force, so this work is included as part of the potential energy term, ΔU . Let's assume the system is released from rest. The weight of mass, m_2 accelerates the system and the potential energy of m_2 is converted into kinetic energy of the system. When mass m_1 has traveled a distance x , mass m_2 has descended the same distance x . So if the initial potential energy of the system is chosen to be zero, the final potential energy of the system is $U_f = -m_2gx$. The initial kinetic energy of the system is zero and the final kinetic energy is $K_f = \frac{1}{2}(m_1 + m_2)v^2$. The work and energy equation becomes:

$$K = \frac{1}{2}(m_1 + m_2)v^2, \quad U = -m_2gx \quad \text{and} \quad 0 = E = K + U = \frac{1}{2}(m_1 + m_2)v^2 - m_2gx$$



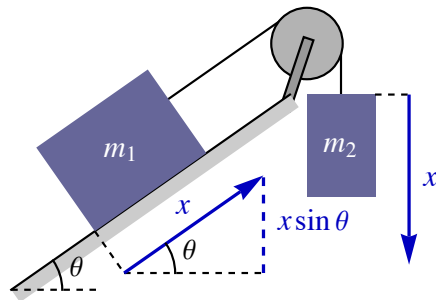
Solving for v^2 gives

$$v^2 = \frac{2m_2g}{m_1 + m_2} x$$

However, if m_1 is on a sloped surface, the change in potential energy of m_1 must be considered. Assuming the system starts from rest, the work and energy equation must now be written as:

$$K = \frac{1}{2}(m_1 + m_2)v^2, \quad U = -m_2gx + m_1(\sin\theta)gx \quad \text{and} \\ 0 = E = K + U = \frac{1}{2}(m_1 + m_2)v^2 - m_2gx + m_1(\sin\theta)gx$$

where θ is the angle that the surface makes with the horizontal.

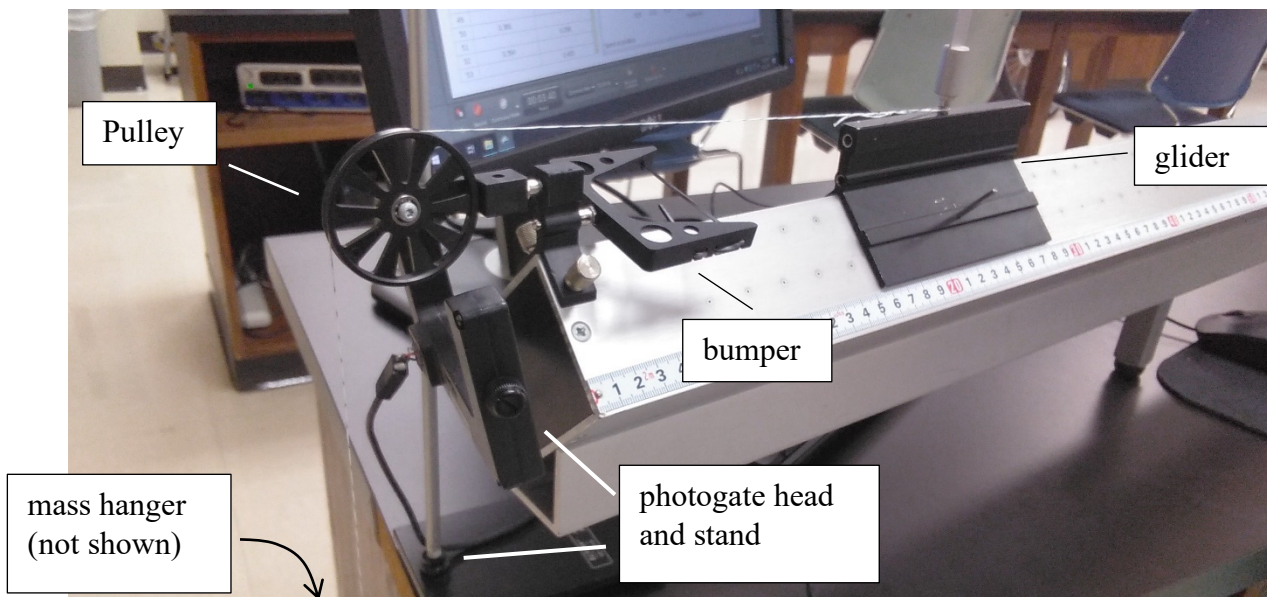


Solving for v^2 gives

$$v^2 = \frac{2(m_2 - m_1(\sin \theta))g}{m_1 + m_2} x$$

Setup:

1. Place the air track on the table and connect the air hose. Set the other end of the track at the edge of the table near the computer and Pasco interface box. Check that the track is level. Place any connector in the top slot of the glider (this will be to hold a string in place). Use a scale to find the mass of the glider. Put the pulley from your Accessory Kit into the upper hole on the end-stop, facing outward. Place a rubber-band bumper in the lower hole of the end-stop, facing inward.



2. Cut a string about 1.5 m long. Tie a loop in one end, attach the other end to a hanger by putting the string through the hole in the hanger and wrapping it around until it catches in the groove. Hang the string over the pulley. Place the photogate stand near the front of the track and adjust the photogate so that it can detect the smart pulley. Plug the photogate into digital input 1 on the interface box and turn the interface on. Slowly spin

the pulley and ensure that the light on the back of the photogate head flashes as it detects the spokes. Start the *Capstone* file called *Energy.cap* in *Physics/Capstone* folder.

Important! When the glider hits the bumper, the air track may be bumped forward. Keep an eye on your system to ensure that the photogate stand does NOT get knocked off the table, and that the photogate head remains aligned to properly detect the pulley.

(A) Energy and systems on a Horizontal Track.

1. Turn the air supply on. Adjust it to a low setting. Place a small amount of mass on the hanger. The *total* hanging mass shouldn't be more than 30g.
2. Pull the glider back to about 60cm away from the end stop. Stop the hanger from swinging.
3. Click Start Recording and then release the glider. Click to stop recording before the glider hits the bumper. If the glider is moving too quickly when it hits the bumper, you may need to stop it by hand.
4. The graph is just for monitoring purposes. If the graph does not look smooth, try again.
5. Once you have a smooth graph, look at the table to the left of the graph. The value v lists the average speed, but the positions x are the starting and ending positions of each block. You will need to compute position for each value of v that you use. The average position is found by averaging the positions before and after the row that the speed is listed in.

$$x_{ave} = \frac{x_i + x_f}{2}$$

(Note that this is an average over position, not time. However, x_{ave} is a better approximation for x than using either x_i or x_f)

Select 10 values of speed v , compute x_{ave} for each and record these values in Table A. Selecting values that are well spread throughout the range of the available data will give better results.

(B) Energy and systems on a Sloped Track.

1. Turn off the air supply. Move the photogate out of the way. Carefully place one support of the track on a book or block so that the end with the pulley is slightly higher than the other end. Replace the photogate and adjust the track and photogate so that the hanging mass, m_2 , has clearance and the photogate can properly monitor the smart pulley.
2. Turn on the air supply. Repeat the process of releasing the glider from rest. Record v and x_{ave} for 10 data points.
3. You will need to be able to compute $\sin \theta$ for your track. Look at the measuring tape on the air track. Use a ruler to measure the vertical height of this measuring tape at the positions 0m and 1m (100cm). These are measurements h_1 and h_2 .

Energy and Systems – Data Sheet

Name _____ Group _____

Fill in the tables. Remember to include proper units at the top of columns. Tables A and B: Use a spreadsheet to plot v^2 vs. x_{ave} . Fit the data with a linear fit and record the slope [experimental]. Compute the theoretical slope. Compare your experimental slope to the theoretical slope by computing a percent error.

Percent error = $\frac{theo-exp}{theo} \times 100\%$. Note: Remember to include the graphs with your lab report !

m_1 _____ kg

Table A. Horizontal track m_2 _____ kg

$$K = \frac{1}{2}(m_1 + m_2)v^2 \quad \text{and} \quad U = -m_2gx$$

set	v ()	x_{ave} ()	v^2 ()	K ()	U ()	$E = K + U$ ()	% diff. between K and $ U $
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

Slope from graph [exp] _____

Slope computed from theory [theo] _____
(show calculations for the slope)

Percent error _____

Show your work. Use the back of this sheet or a separate sheet if you need more space

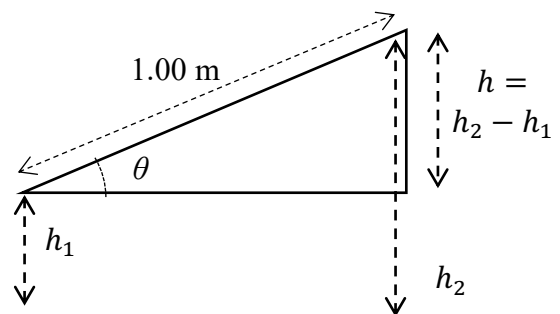
Table B. Sloped track m_2 _____ kg

$$K = \frac{1}{2}(m_1 + m_2)v^2 \quad \text{and} \quad U = -m_2gx + m_1(\sin\theta)gx$$

set	v ()	x_{ave} ()	v^2 ()	K ()	U ()	$E = K + U$ ()	% diff. between K and $ U $
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

Other data for part B

h_1 (m)	h_2 (m)	$\sin \theta$ [computed]



Slope from graph [exp] _____

Slope computed from theory [theo] _____
(show all calculations for the slope)

Percent error _____

Question 1 What are the units of the slope of the graph?

Question 2 Does your data in part A support conservation of energy? Explain. Because $E = K + U = 0$, we should have $K = |U|$. Were they close?

Question 3 Compute the experimental acceleration for the system from your data or graphs for Part A and Part B. The equation $v^2 = v_0^2 + 2a\Delta x$ may be of help.

Part A:

Part B:

Question 4 Does your data in part B support conservation of energy? Explain. Because $E = K + U = 0$, we should have $K = |U|$. Were they close?

Question 5 How would the results of this experiment be different if the air supply were turned off and there were friction? What would happen to the values of E as the x_{ave} values progressed?

Conservation of Linear Momentum

Equipment and Setup: Air track, air supply with hose, Photogates (2), Gliders (2), Accessory kit, Capstone file – Linear Momentum.cap

Theory

There are four fundamentally conserved quantities in physics: energy, linear momentum, angular momentum and electric charge. This experiment studies the conservation of linear momentum in one-dimensional collisions.

Momentum is the vector quantity defined as the mass multiplied by the velocity: $\vec{p} = m\vec{v}$. The momentum form of Newton's second law for a particle is $\vec{F}_{\text{net}} = d\vec{p}/dt$. A collection of particles is called a system of particles. To generalize the second law for a particle to that for a system, we break up all the forces into internal and external forces, where internal forces are between particles in the system and external forces are the forces of everything outside the system acting on the particles of the system. When the second law expressions are summed over all particles in the system, the internal forces cancel because of Newton's third law. This gives a simple result that the bulk motion of the system is determined only by the external forces.

$$\vec{F}_{\text{net}}^{\text{ext}} = \frac{d\vec{p}_{\text{tot}}}{dt}$$

where $\vec{F}_{\text{net}}^{\text{ext}}$ is the sum of all the external forces on all the particles of the system and \vec{p}_{tot} is the total momentum of the system, the sum of all the momenta. The conservation of momentum follows from the second law for a system. When the net external force on a system vanishes, the total momentum of the system is conserved.

$$\vec{F}_{\text{net}}^{\text{ext}} = 0 \Rightarrow \Delta\vec{p}_{\text{tot}} = 0 \text{ or } \vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f}$$

For two-body collision problems when there are no external forces the total momentum is conserved, since the forces between the two bodies are internal forces. In this experiment we will study one-dimensional two-body collisions using an air track. We will treat one-dimensional vectors as real numbers, where the sign corresponds to the direction. It is essential in this experiment to be very careful with signs. Conservation of momentum becomes

$$p_{\text{tot},i} = p_{\text{tot},f} \Rightarrow m_1v_{1,i} + m_2v_{2,i} = m_1v_{1,f} + m_2v_{2,f}$$

The relevant energy is kinetic energy. Generally, energy is not conserved in a collision. When kinetic energy is conserved, we say that the collision is *elastic*.

$$K_{\text{tot},i} = K_{\text{tot},f} \Rightarrow \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2 \quad (\text{elastic})$$

A collision is *inelastic* when, not surprisingly, it is not elastic.

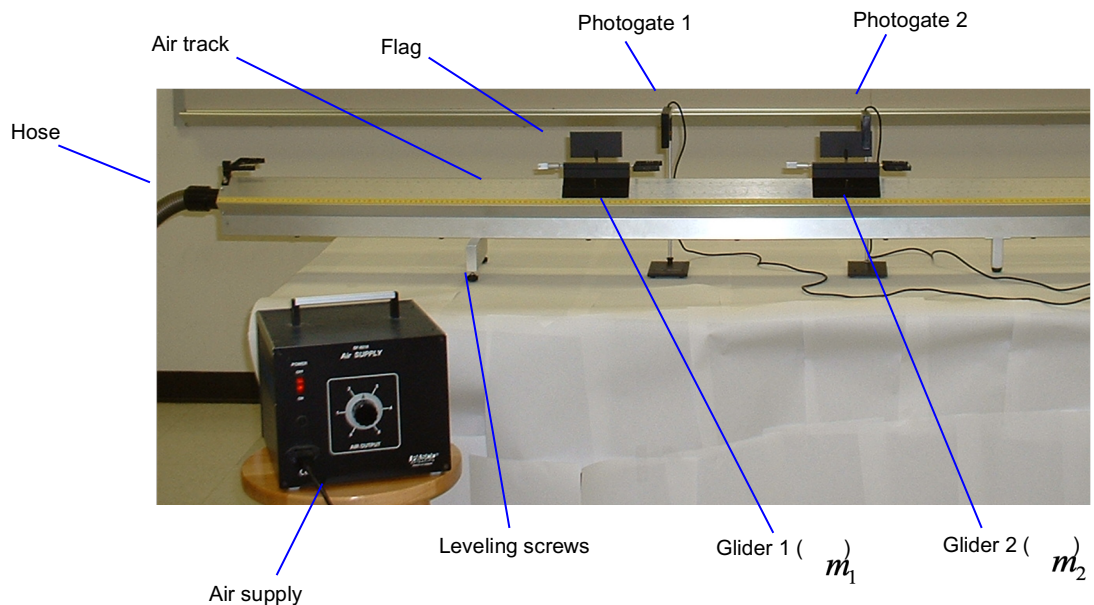
$$K_{\text{tot},i} \neq K_{\text{tot},f} \quad (\text{inelastic})$$

There is an important special case of inelastic collisions where the two bodies lose as much energy in the collision as is possible, while still conserving momentum. That occurs when the two bodies stick together and move off as one. With that there is a single final velocity $\vec{v}_f = \vec{v}_{1,f} = \vec{v}_{2,f}$. This condition is known as a totally or *completely inelastic* collision. For one-dimension totally inelastic collisions the conservation of momentum becomes.

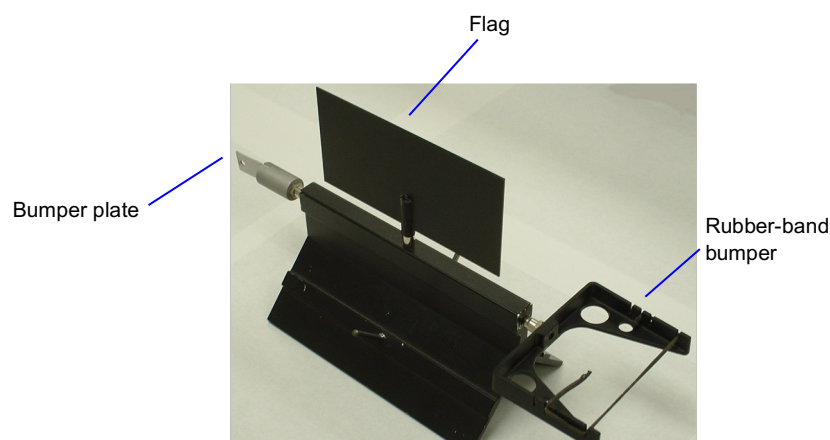
$$p_{\text{tot},i} = p_{\text{tot},f} \Rightarrow m_1v_{1,i} + m_2v_{2,i} = (m_1 + m_2)v_f \quad (\text{totally inelastic})$$

Procedure

Setup:



1. Gather the equipment listed at the top of the experiment: an air supply, hose, two gliders, an accessory kit and two photogates. *Be very careful moving the air track, it is heavy and bulky and you could easily hurt yourself or someone else if you carry it carelessly.*
2. Plug in the air supply. The air supply can either be placed on the lab table or on the floor below it. Connect the air supply to the air track with the hose.
3. Set up the glider using parts in your accessory kit as shown below. The flag shown below may be wider than the flags you are provided. The rubber bands of the bumper will create collisions that are elastic to



a good approximation. The bumper plate must be added to provide a counterweight to the bumper.

4. The assumption that the net external force acting on the system is zero is only approximate and this will be a source of error. There will be some friction, some random force due to the air blowing by the gliders and most importantly gravity. To minimize the effect of gravity, the air track must be as level as possible. To level the air track, turn on the air supply and place a properly configured glider at its center.

The glider will float on the blowing air; this creates a nearly frictionless situation. Adjust the levelling screws on the air track until the glider does not move one way or the other with a bias.

- Place the photogates above the table so the flags of the glider break the beam of the photogate as they pass through. Have gate 1 to the left of gate 2. Plug gate 1 into Digital Channel 1 and gate 2 into channel 2 on the Pasco interface box. From the Physics folder on the desktop open the Capstone file Linear Momentum.cap. Hit record and watch what happens when you block and unblock the photogates with your hand. There are two tables: Table 1 shows the total time gate 1 was blocked each time it was blocked and Table 2 similarly refers to gate 2. Keep the two photogates sufficiently spaced so that the collision happens entirely between the photogates. If they are too far apart, there will be a larger error due to friction, the random effects of the blowing air and the small amount that the air track is off-level.
- For each part of the experiment you will measure the masses using a scale in the lab room. Take the masses of the two gliders to be the same initially and when adding weight then add the appropriate number of 50 gram weights symmetrically.

Calculating Velocities, Momenta and Kinetic Energies

- Measure the width of the flags. This is your value of Δx ; record that value in your Data Sheet. This Δx will stay the same throughout the experiment.
- The tables in Capstone give the Δt values; the time each gate is blocked every time it is blocked.

To find the velocity from this information, it is:

$$v = \pm \frac{\Delta x}{\Delta t}$$

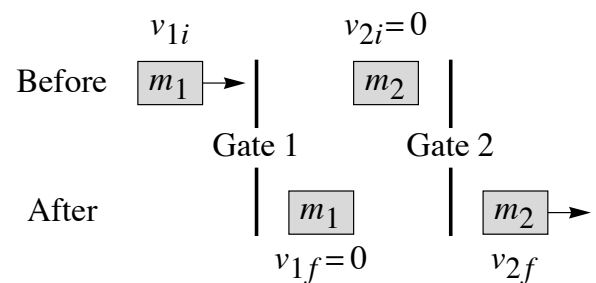
That is, the glider moves a distance Δx , one flag width, in the recorded time Δt . The sign is critical. The photogate does not know the direction of the glider’s velocity; you must put this direction in by hand by including the correct sign. The convention we will use is: v is positive when moving to the right and negative when moving to the left. Remember that gate 2 is to the right of gate 1.

- The data for each part of this experiment will be the Δt values. The time subscripts in the data tables refer to the corresponding velocities. For example, $\Delta t_{1,f}$ refers to the time needed to calculate $v_{1,f}$ but this value could be from Table 2 for gate 2, if glider 1 moves through gate 2 after the collision.
- To calculate the momentum and kinetic energy for each part of the experiment use these expressions with the appropriate initial and final subscripts.

$$p_{tot} = m_1 v_1 + m_2 v_2 \quad \text{and} \quad K_{tot} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Part (A) – Elastic, $m_1 = m_2$, $v_{2i} = 0$, $v_{1f} = 0$

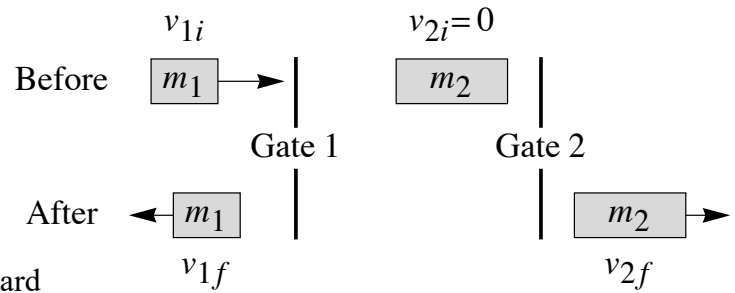
Take the two masses to be the same and record that value. The rubber band should be on to make the collision elastic. Begin with m_2 at rest between the two photogates. m_1 should start before gate 1. Give it a small push toward m_2 . Place the gliders and photogates so that m_1 has fully passed through gate 1 before hitting m_2 . m_1 should come to a stop and m_2 will pass through gate 2.



Conservation of Linear Momentum

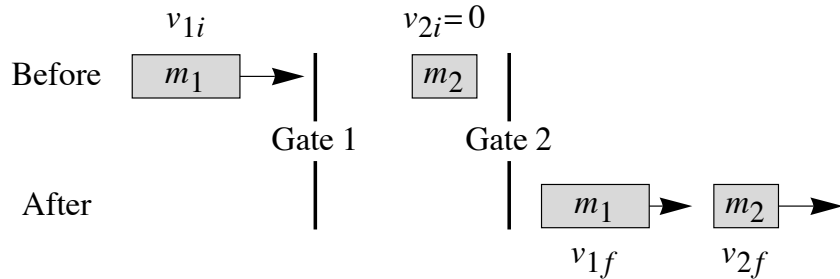
Part (B) – Elastic, $m_1 < m_2$, $v_{2i} = 0$

Use the same m_1 as Part (A) but add additional weights to m_2 . Record both masses. When adding weight do it symmetrically, the same on both sides of the glider. The rubber band should still be on for an elastic collision. Initially position the gliders as before with m_2 at rest. Give m_1 a small push toward m_2 . Now m_1 should bounce backward passing through gate 1 a second time and m_2 will pass through gate 2.



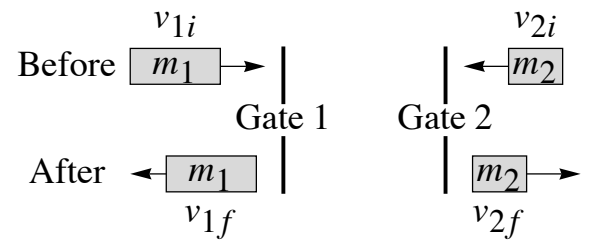
Part (C) – Elastic, $m_1 > m_2$, $v_{2i} = 0$

Now move the additional weights from m_2 to m_1 . Record both masses. Again give m_1 a small push toward m_2 . Now m_1 should continue moving forward and pass through gate 2 after m_2 .



Part (D) – Elastic, $m_1 > m_2$, Both moving

Now both masses start outside the gates with m_1 larger than m_2 . The same person should give both masses a light push toward each other. If you hear a noise when they collide, they were pushed too hard and jumped off the track when colliding. If so, redo it. For this part the two photogates should be a bit further apart, so the entire collision can happen entirely between them.



Part (E) – Totally inelastic, $m_1 = m_2$, $v_{2i} = 0$

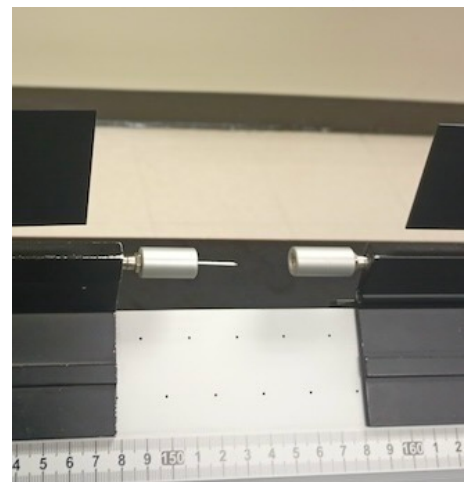
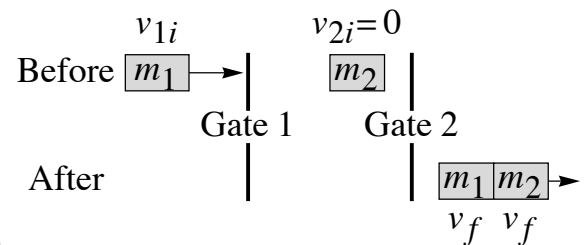
Remove the rubber band and bumper plate from the two masses and replace them with the pin and wax-filled cylinder as shown in the picture. This arrangement will cause the two masses to stick together to make the collision totally inelastic. Give m_1 a push toward m_2 , initially at rest. After they stick together both will move through gate 2. Since gate 2 will be blocked twice the two times should be close to the same. Choose the first time to find the single final velocity. For this part you will find the fraction of (kinetic) energy lost. That fraction is

$$\frac{K_i - K_f}{K_i}$$

You will show theoretically in Problem 2 that this fraction of energy lost is equal to

$$\frac{m_2}{m_1 + m_2}$$

Here you will calculate both values and compare them by finding the percent error, taking the theoretical result as the accepted value.



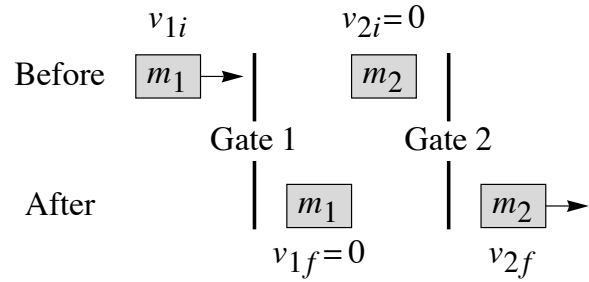
Linear Momentum – Data Sheet

Name _____ Group _____

(A) Elastic, $m_1 = m_2$, $v_{2i} = 0$, $v_{1f} = 0$

$m_1 = m_2 =$ _____ kg

Trial	Δt_{1i}	Δt_{2f}
1		
2		

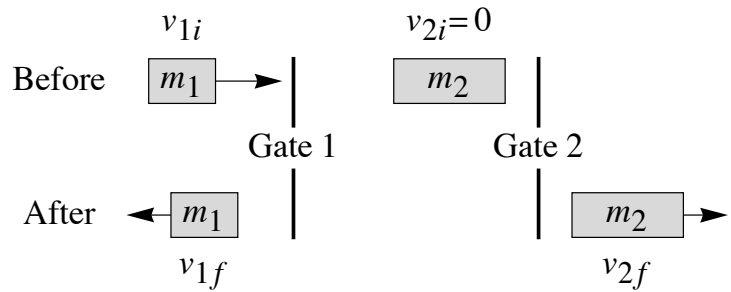


Trial	v_{1i}	v_{2f}	$p_{tot,i}$	$p_{tot,f}$	% diff. p_{tot}	$K_{tot,i}$	$K_{tot,f}$	% diff K
1								
2								

(B) Elastic, $m_1 < m_2$, $v_{2i} = 0$

$m_1 =$ _____ kg $m_2 =$ _____ kg

Trial	Δt_{1i}	Δt_{1f}	Δt_{2f}
1			
2			

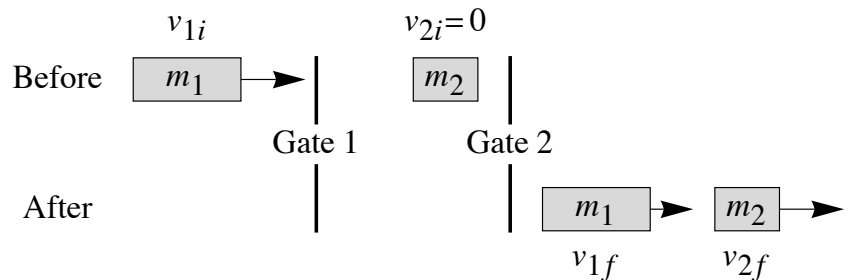


Trial	v_{1i}	v_{1f}	v_{2f}	$p_{tot,i}$	$p_{tot,f}$	% diff. p_{tot}	$K_{tot,i}$	$K_{tot,f}$	% diff K
1									
2									

(C) Elastic, $m_1 > m_2$, $v_{2i} = 0$

$m_1 =$ _____ kg $m_2 =$ _____ kg

Trial	Δt_{1i}	Δt_{1f}	Δt_{2f}
1			
2			



Problem 1 For elastic one dimensional collisions: $v_{1i} + v_{1f} = v_{2i} + v_{2f}$. Verify the accuracy of this *for both trials* of part (D). Compare $v_{1i} + v_{1f}$. and $v_{2i} + v_{2f}$. Give their % difference.

Trial	$v_{1i} + v_{1f}$	$v_{2i} + v_{2f}$	% difference
1			
2			

Problem 2 Show theoretically that for a totally inelastic collision with m_2 initially at rest that:

$$\frac{K_i - K_f}{K_i} = \frac{m_2}{m_1 + m_2} \quad (\text{Hint: Write the kinetic energy as } K = \frac{p^2}{2m} \text{ and use the fact that } p_i = p_f.)$$

Conservation of Angular Momentum (“A” Base Platform)

Equipment and Setup: Smart Pulley, “A” Base Rotational Platform with Photogate, Bar, Center Post, Capstone file – Angular Momentum - A Base.cap

(A) Sliding Mass on a Bar

Setup

- Remove the disk and ring. Place the bar and center post on the platform.



- Remove the photogate from the smart pulley and attach to the small steel bar at the base of the platform. Adjust its height so that the bar rotates freely.
- Level the platform.
 - Keep the heavy sliding mass far from the center of the bar so that it is not balanced.
 - There are two screws on the base for leveling. Turning clockwise raises that side and turning counterclockwise lowers it.
 - Align the bar to be parallel to the leveling screws. Adjust the screws together in the same sense (both clockwise or counterclockwise) until the bar stays parallel.
 - Then align the bar perpendicular to the screws and rotate the screws together but in the opposite sense (clockwise for one and counterclockwise for the other) until it balances.
 - Repeat the previous two steps with smaller adjustments until the bar is level in both orientations.

Data

- Measure and record the length and width of the bar, L and W . Because the heavy pivot at the center of the bar's base contributes negligibly to its moment of inertia, the value of the mass of the bar without the heavy pivot at its center was premeasured and is given below as M .
- Measure and record the mass of the sliding mass, m . There is a second identical mass, which you will not use in the experiment, but can use it to find m .

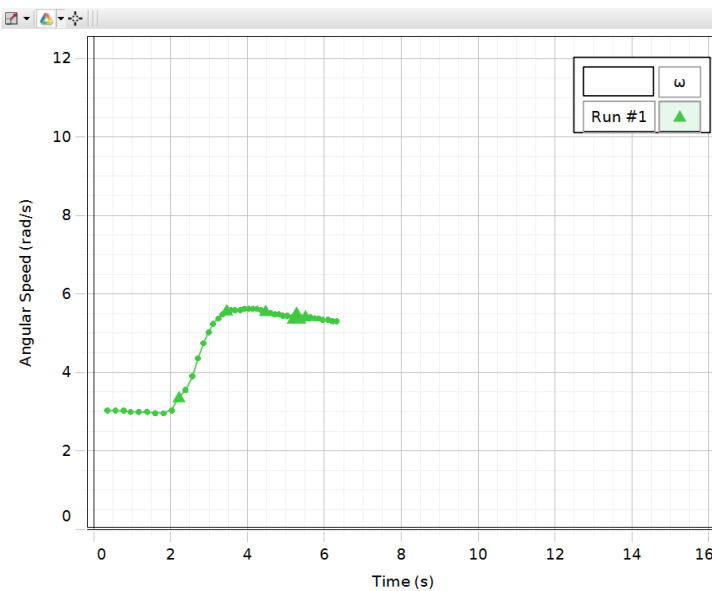
- Set the stopper to a position far from the center. Push the sliding mass against it and record the position of its center, x_i . It is difficult to identify the center of the sliding mass, so find its midpoint by averaging the positions of the sides. You may also move the stopper a bit to make it easier to identify the midpoint.
- Pull the mass all the way toward the center until it is against the pulley. Find the position of its center as you did in the previous step; this position is x_f .
- Put the sliding mass against the stopper. Pass the attached string around the pulley and up the center post, so it can be pulled.
- Start the bar rotating and start recording data. While rotating, pull the sliding mass toward the center. After hitting the pulley near the center, stop recording.
- Read the initial and final angular velocities off the Capstone graph and record your results in the table.
- Repeat this twice for a total of three trials.

Graph Display Icons

To refresh the graph choose the "Scale Axes" icon.

The second icon selects different runs.

To find the coordinates of a data point click the "Show Coordinates" icon. Drag this (from the box at the center) to the desired data points and record the angular velocities. To delete, right-click the box and choose "Delete tool"



ω vs. t

Bar: $M = 0.5232$ kg $L =$ _____ m $W =$ _____ m

Sliding Mass: $m =$ _____ kg $x_i =$ _____ m $x_f =$ _____ m

Trial 1		Trial 2		Trial 3	
ω_i (rad/s)	ω_f (rad/s)	ω_i (rad/s)	ω_f (rad/s)	ω_i (rad/s)	ω_f (rad/s)

Calculations

- Calculate I_{bar} the moment of inertia of the bar. Include the formula you use for this calculation. Treat the bar as a uniform rectangular plate. The lack of uniformity across the width of the bar will introduce a negligible error.
- Treating the sliding mass as a point mass at either x_i or x_f from the center, find I_i and I_f , the moments of inertia of the bar with sliding mass before and after you drag it. These moments will be the sum of the moment of the bar and the moment of a point mass a distance of x_i or x_f from the center.
- Using $L = I\omega$, calculate the initial and final angular momenta L_i and L_f . Theoretically these should be equal but your values will differ due to experimental error. Calculate the percent differences between your experimental values. The percent difference for two experimental values is: $\frac{|\text{difference}|}{\text{average}} \times 100\%$.
- Calculate the initial and final kinetic energies, where $K = \frac{1}{2}I\omega^2$. Also, calculate the energy gain ΔK . I_{bar} :

$I_{\text{bar}} = \underline{\hspace{2cm}} \text{ kg m}^2$

Show formulas and work.

I_i and I_f : $I_i = \underline{\hspace{2cm}} \text{ kg m}^2$ $I_f = \underline{\hspace{2cm}} \text{ kg m}^2$

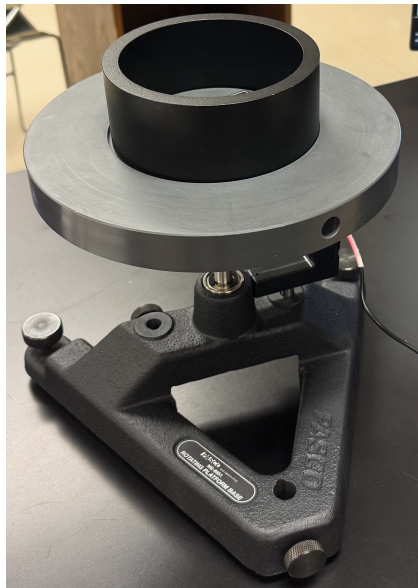
Show formulas and work.

Trial	L_i	L_f	% Diff.	K_i	K_f	$\Delta K = K_f - K_i$
1						
2						
3						

(B) Rotational Collisions

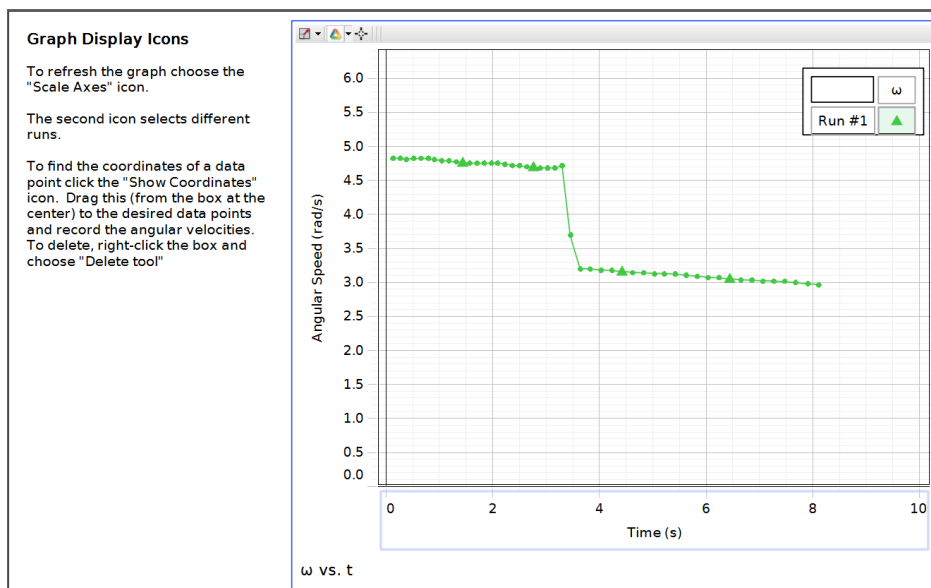
Setup

- Remove the bar and center post and place the disk and ring back on the platform. There is a slot on the platter to accept the ring; keep this on top.



Data

- Measure and record the mass and diameter of the disk, m_1 and d_1 .
- Measure and record the mass and average diameter of the ring, m_2 and d_2 . The average diameter is the average of its inside and outside diameters. A simple way to find this is to measure the diameter from the inside edge of one side to the outside edge of the other.
- With the ring removed, start the disk rotating and start recording data. While rotating, drop the ring onto the disk so that it lands in the slot. Stop recording.
- Read the initial and final angular velocities off the Capstone graph and record your results in the table.
- Repeat this twice for a total of three trials.



Disk: $m_1 =$ _____ kg $d_1 =$ _____ m

Ring: $m_2 =$ _____ kg $d_2 =$ _____ m

Trial 1		Trial 2		Trial 3	
ω_i (rad/s)	ω_f (rad/s)	ω_i (rad/s)	ω_f (rad/s)	ω_i (rad/s)	ω_f (rad/s)

Calculations

- Calculate I_1 , the moment of inertia of the disk and I_2 , the moment of inertia of the ring. Include the formulas you use for these calculations. Be careful to not use diameters as radii in the formulas.
- Before the ring is dropped, the *initial* case, there is only the disk with moment I_1 rotating with angular velocity ω_i . After the ring is dropped, the *final* case, both the disk with moment I_1 and the ring with moment I_2 are rotating together with angular velocity ω_f .
- As in Part (A) calculate the initial and final angular momenta, L_i and L_f , and calculate their percent differences. Also, calculate the initial and final kinetic energies, K_i and K_f ,
- The fraction of energy lost is $\frac{K_i - K_f}{K_i}$. In the Conservation of Linear Momentum experiment we showed that for the case of a totally inelastic collision with m_2 initially at rest, the fraction of energy lost is $\frac{m_2}{m_1 + m_2}$. Here we have the rotational analog of that, so the fraction of energy lost here should be $\frac{I_2}{I_1 + I_2}$.
- Calculate the percent error between $\frac{K_i - K_f}{K_i}$ and $\frac{I_2}{I_1 + I_2}$, where $\frac{I_2}{I_1 + I_2}$ is the theoretical result. The percent error between an experimental value and a theoretical value is: $\frac{|experimental - theoretical|}{theoretical} \times 100\%$.

Disk: $I_1 =$ _____ kg m²

Ring: $I_2 =$ _____ kg m²

Trial	L_i	L_f	% Diff.	K_i	K_f	$\frac{K_i - K_f}{K_i}$	$\frac{I_2}{I_1 + I_2}$	% Error
1								
2								
3								

Show formulas and work.

Questions

A-1 Does your data in Part (A) support conservation of angular momentum?

A-2 Kinetic energy should not be conserved in Part (A) and the energy gain ΔK should be positive. Where does the energy come from?

A-3 Suppose in calculating the moment of inertia you used the formula for a thin rod of length L , and not that for an $L \times W$ rectangular plate. What percent error would be introduced in the calculated value of the moment of the bar, taking the rectangular plate as the accepted or theoretical value? Would the moment be larger or smaller than the accepted value?

B-1 Does your data in Part (B) support conservation of angular momentum?

B-2 Kinetic energy in part (B) should not be conserved. Where does the energy lost go?

Orbits and Kepler's Laws

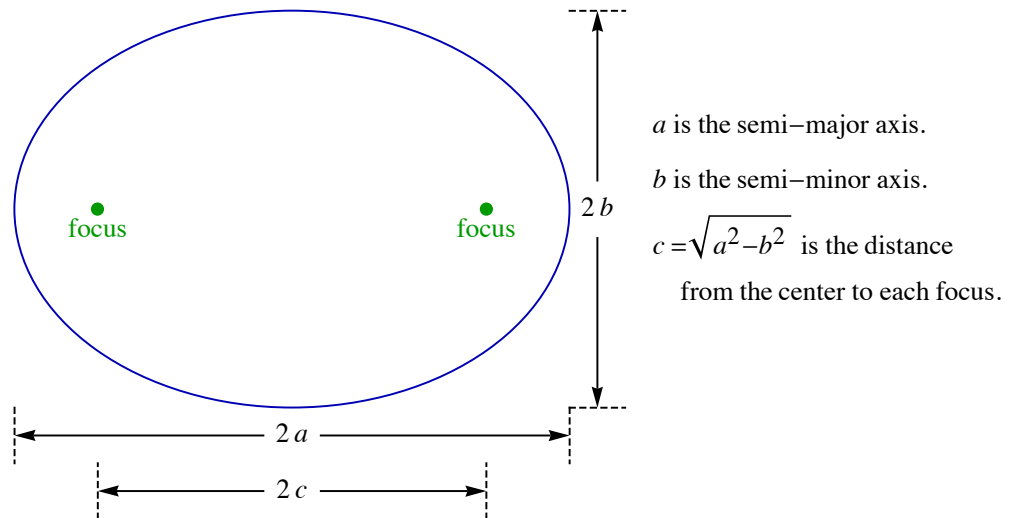
Equipment and Setup: Mathematica file – Kepler.nb

Theory

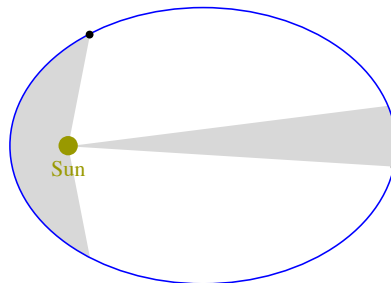
Kepler's Laws

Kepler found three laws describing planetary orbits about the sun. He did his work after Copernicus and before Newton. The three laws are:

1. Planets move in elliptical orbits about the sun, which is at one of the focal points of the ellipse.



2. A radial line from the sun to a planet sweeps out equal areas in equal time intervals.



3. The orbital period squared is proportional to the semi-major axis cubed.

$$T^2 \propto a^3$$

General Orbital Dynamics

The gravitational force on mass m due to a second mass M is

$$\vec{F} = -\frac{GMm}{r^2} \hat{r}$$

where M is at the origin, \vec{r} is the vector from M to m and $\hat{r} = \vec{r}/r$ is the unit vector in the direction of \vec{r} .

We will assume that M is much larger than m ($M \gg m$) so that M stays fixed at the origin. The inverse square law shown above is not the only possibility. A central force is one that points toward the sun. More generally, we can imagine some central force that is attractive and decreases with distance, going to zero at infinity. Here, we will consider only central forces that are inverse power laws, meaning that they drop off as the inverse of r to a power p , $F \propto 1/r^p$

$$\vec{F}_p = -\frac{GMm}{r^p} \hat{r} = -F_p \hat{r}.$$

Since we are interested in orbits about the sun, we take M to be the mass of the sun. m is the planet's mass which, since $\vec{a} = \vec{F}/m$ does not involve the kinematics of the orbit. A very useful choice of units will simplify our discussion; measure all lengths in astronomical units, AU, where 1 AU is the radius of the earth's orbit and measure time in years. In these units, the speed of a circular orbit of radius 1 AU with period 1 year is

$$r = 1 \text{ AU and } T = 1 \text{ year} \Rightarrow v = \frac{2\pi r}{T} = 2\pi \frac{\text{AU}}{\text{year}} \text{ or } v = 2\pi \text{ (in Solar System Units)}$$

To simplify the expressions, we will omit the units and just refer to this as Solar System Units. For circular orbits, since $F_p/m = a_c$, we have

$$\frac{GM}{r^p} = a_c = \begin{cases} \frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r^{p-1}} = \frac{4\pi^2}{r^{p-1}} \\ \left(\frac{2\pi}{T}\right)^2 r \Rightarrow T^2 = \frac{4\pi^2}{GM} r^{p+1} = r^{p+1} \end{cases} \quad (\text{in Solar System Units})$$

To simplify the two expressions above we used that in Solar System units, the earth's circular orbit has $v = 2\pi$ and $r = 1$. This gives $GM = 4\pi^2$; we inserted that into the expressions above at the right.

Newtonian Gravity

Newton was able to show that for central forces, the inverse square law $p = 2$ uniquely gives Kepler's laws. To show this mathematically is beyond the scope of this course but we may demonstrate this using a computer simulation.

Computer Setup

- This orbit simulation has a planet or satellite initially at a distance r_0 from the sun with an initial velocity of magnitude v_0 perpendicular to the radial direction.
- The interactive panel has two tabs at the left. The “Other Power Laws” tab gives a central force proportional to the inverse of the p^{th} power of r , $F \propto 1/r^p$; under this tab one can choose any value of p .
- The “Newtonian Gravity” tab gives the physically correct case of $p = 2$, with the additional controls and orbit information. There are three buttons listed as “Display Swept Area?”. The “Static with value” button shows a box to choose a value of Δt ; above the graphic it gives a slider to vary the value of t , the starting time of the interval, and lists the value of the area. The “Animated” button shows the area in an animation. Still under the “Newtonian Gravity” tab, to the right of these buttons is orbit data.
- To the right of the tab area are three checkboxes. “Animate” opens animation controls above the graphic. (Note that the “Animate” checkbox cannot be selected when the “Display Swept Area?” button is chosen under the “Newtonian Gravity” tab.) “Show r_0 and v_0 ” displays the initial position and the initial velocity, which is perpendicular to the radial line from the sun. “Show Earth Orbit” shows the earth’s orbit as a reference.
- There are three input fields at the right of the controls to vary the initial values of v_0 and r_0 , and to vary the stopping time t_{max} .

Section A: Other Power Laws

Use the “Other Power Law” tab for this section.

1. The simplest power law is where gravity is inversely proportional to the distance from the sun. This is the $p = 1$ case. Set the value of r_0 to 1AU. Use $v_0 = 1 \times 2\pi$ AU/yr and describe the orbit.
2. Keeping $p = 1$ and $r_0 = 1\text{AU}$ set $v_0 = 0.5 \times 2\pi$ AU/yr. Describe the results. Is the orbit closed?
3. Keeping $p = 1$ and $r_0 = 1\text{AU}$ set $v_0 = 1.5 \times 2\pi$ AU/yr. Choose a larger value of t_{max} if needed. Describe the results. Is the orbit closed?

4. Now consider the $p = 3$ case, an inverse cube law. Set the value of r_0 to 1AU. Use $v_0 = 1 \times 2\pi$ AU/yr and describe the orbit.
5. Keeping $p = 3$ and $r_0 = 1$ AU set the speed just under that for a circular orbit. Use $v_0 = 0.99 \times 2\pi$ AU/yr. Describe the Orbit.
6. Keeping $p = 3$ and $r_0 = 1$ AU set the speed just over that for a circular orbit: $v_0 = 1.01 \times 2\pi$ AU/yr. Describe the results.
7. Now look at the $p = 2$ case, the inverse square law. Set the value of r_0 to 1AU and use $v_0 = 0.5 \times 2\pi$ AU/yr. This gives Kepler's closed elliptical orbit. With slight deviations from the inverse square law we would expect elliptical orbits that do not quite close. Keeping the same values of v_0 and r_0 try $p = 1.98$ and $p = 2.02$. Describe the results.

Section B: Kepler's Laws

Use the "Newtonian Gravity" tab for this section.

1. Use $r_0 = 1$ AU, $v_0 = 0.5 \times 2\pi$ AU/yr and $t_{max} = 1$ yr. Record r_{min} , r_{max} and T .

$$r_{min} = \text{_____ AU}, \quad r_{max} = \text{_____ AU}, \quad T = \text{_____ yr},$$

Calculate the semi-major axis a , then find a^3 and T^2 . With these units Kepler's third law implies the values of a^3 and T^2 should be equal. Compare them by finding their percent difference.

$$a = \text{_____ AU}, \quad a^3 = \text{_____ AU}^3, \quad T^2 = \text{_____ yr}^2, \quad \% \text{ diff.} = \text{_____}$$

2. Use $r_0 = 1\text{AU}$, $v_0 = 0.5 \times 2\pi \text{ AU/yr}$ and $t_{\text{max}} = 1 \text{ yr}$. In the “Display Swept Area?” buttons choose “Animated”. Set $\Delta t = 0.05\text{yrs}$ and click the *play* button above the graphic. Describe what you observe.

3. Keeping the same values of r_0 and v_0 choose $t_{\text{max}} = 0.5 \text{ yr}$. In the “Display Swept Area?” buttons choose “Static with value”. Using the same value $\Delta t = 0.05\text{yrs}$ record the value of Area at $t = 0 \text{ yr}$. With the slider adjust t until the swept area is nearest the sun and record t and Area.

$$t = 0 \text{ yrs} \Rightarrow \text{Area} = \underline{\hspace{2cm}} \text{ AU}^2, \quad t = \underline{\hspace{2cm}} \text{ yr} \Rightarrow \text{Area} = \underline{\hspace{2cm}} \text{ AU}^2$$

Discuss why the two area values are equal.

4. Halley’s comet has a aphelion distance of $r_{\text{max}} = 35\text{AU}$ and a perihelion distance of $r_{\text{min}} = 0.59\text{AU}$. Begin with $r_0 = 35\text{AU}$, this will become r_{max} . For “Display Swept Area?” choose “None”. Choose different small values of v_0 until you get the correct value of r_{min} . Set a long t_{max} to guarantee a full orbit; $t_{\text{max}} = 100 \text{ yr}$ will suffice. Values of r_{min} between 0.55 and 0.63 AU will be close enough. (You must be systematic about this; if the orbit is unbound or if r_{min} is too large then choose a smaller v_0 value and if r_{min} is too small choose a larger v_0 .) Record r_{min} , v_0 and the period.

$$r_{\text{min}} = \underline{\hspace{2cm}} \text{ AU}, \quad r_{\text{max}} = 35 \text{ AU}, \quad v_0 = \underline{\hspace{2cm}} \times 2\pi \text{ AU/yr}, \quad T = \underline{\hspace{2cm}} \text{ yr},$$

5. The escape speed at some distance from the sun is $\sqrt{2}$ times that for a circular orbit. Choose $r_0 = 1\text{AU}$ and $v_0 = \sqrt{2} \times 2\pi \text{ AU/yr}$. (Input Sqrt[2] into the field.) Take a large value of t_{max} : $t_{\text{max}} = 30 \text{ yr}$. What is the shape of the orbit at this critical value?

6. Now consider a speed larger than this critical escape speed. Choose $r_0 = 1\text{AU}$, $v_0 = 1.5 \times 2\pi \text{ AU/yr}$ and $t_{\text{max}} = 30 \text{ yr}$. What is the shape of this orbit?

The Period of a Simple Pendulum

Equipment and Setup: Support rod and base, Pendulum Clamp, Pendulum bob set, string, Meter stick, Protractor, Photogate, Capstone file – Pendulum.cap

Theory

Simple harmonic motion (SHM) occurs whenever the ordinary differential equation for x as a function of t has the form:

$$\frac{d^2x}{dt^2} = -\omega^2x$$

In this expression ω is known as the angular frequency. The standard example of this is the mass-spring system, where Newton's second law gives the differential equation and then an expression for ω can be read off.

$$F_{\text{net}} = ma \Rightarrow -kx = m \frac{d^2x}{dt^2} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

The general solution of this simple harmonic motion differential equation is:

$$x(t) = A \cos(\omega t + \phi)$$

Where the amplitude A is the largest distance from equilibrium ($x = 0$) and the phase angle ϕ is related to shifts in time. The time for each full oscillation is the period T which is related to ω .

$$T = \frac{2\pi}{\omega}$$

Note that the period is independent of the amplitude A . If a particle is released from rest, then the initial distance from equilibrium is the amplitude A and the phase angle is zero, $\phi = 0$.

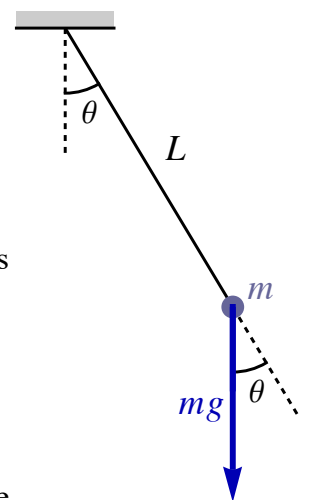
The simple pendulum consists of a small bob of mass m hanging at the end of a string of length L that is fixed at the top. The bob and string are pulled to a maximum angle of θ_0 from vertical and released from rest. We will find the differential equation for the angle as a function of time using the rotational second law: $\tau_{\text{net}} = I\alpha$. The net torque is the torque due to gravity. Since $\tau = rF \sin \theta$, the net torque becomes: $\tau_{\text{net}} = \tau_{\text{grav}} = -Lmg \sin \theta$. The reason for the minus sign is that by the definition of θ , counterclockwise is positive and the torque gives a clockwise rotation. The moment of inertia about the fixed point of the string is $I = mL^2$. The angular acceleration α is the second time derivative of the angle θ . This gives the differential equation.

$$\tau_{\text{net}} = I\alpha \Rightarrow -Lmg \sin \theta = mL^2 \frac{d^2\theta}{dt^2}$$

Defining $\omega = \sqrt{g/L}$ we can write the differential equation for the angle as a function of time.

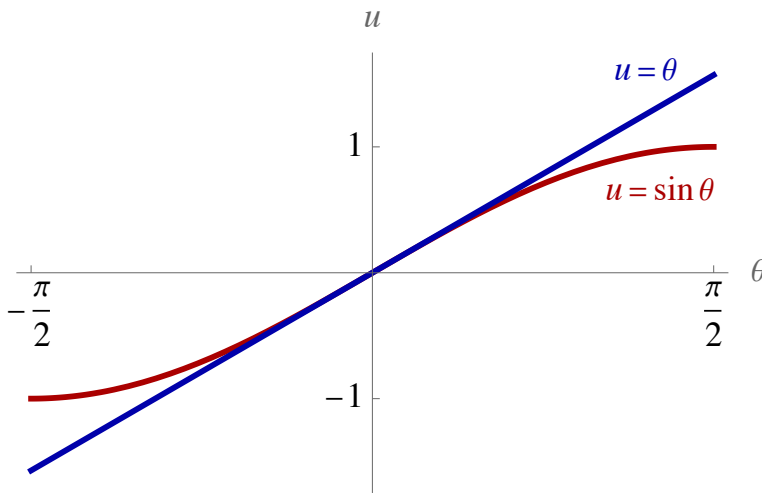
$$\frac{d^2\theta}{dt^2} = -\omega^2 \sin \theta \quad \text{where } \omega = \sqrt{\frac{g}{L}}$$

This is not quite of the form of the SHM differential equation, since the second derivative of the angle is proportional to the sine of the angle and not the angle itself. From the Taylor (Maclaurin) expansion of the sine function we can see that for sufficiently small angles, the sine of the angle is approximately equal to the angle in radians.



$$\sin \theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \dots \Rightarrow \sin \theta \cong \theta$$

We can see the quality of this approximation with the graph and the table below.



θ in degrees	θ in radians	$\sin \theta$
1	0.0174533	0.0174524
2	0.0349066	0.0348995
5	0.0872665	0.0871557
10	0.174533	0.173648
20	0.349066	0.34202
30	0.523599	0.5
45	0.785398	0.707107
60	1.0472	0.866025
90	1.5708	1

Define θ_0 as the angle amplitude of the pendulum's oscillation, the largest angle from vertical. For small-angle amplitude oscillations, the simple pendulum is an example of simple harmonic motion and the period is:

$$T_0 = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}} \quad \text{(small-angle period)}$$

This period is independent of the amplitude θ_0 as long as the amplitude stays small. For larger amplitude oscillations, however, the period does vary with the amplitude. It turns out that there is not a closed-form expression for the period, where a closed form expression is one that can be written in terms of standard known functions. It is common to write such expressions as an infinite series. A series solution for the period can be found; it is:

$$T = \frac{2\pi}{\omega} \sum_{p=0}^{\infty} a_p \sin^{2p} \left(\frac{\theta_0}{2} \right) \quad \text{where} \quad a_p = \left(\frac{(2p-1)!!}{(2p)!!} \right)^2$$

where the double factorial is defined by $n!! = n \times (n-2)!!$, taking $0!! = 1 = 1!!$; giving $7!! = 7 \cdot 5 \cdot 3 \cdot 1$ and $8!! = 8 \cdot 6 \cdot 4 \cdot 2$. The first few terms of the series are

$$T = \frac{2\pi}{\omega} \left(1 + \frac{1^2}{2^2} \sin^2 \left(\frac{\theta_0}{2} \right) + \frac{1^2 3^2}{2^2 4^2} \sin^4 \left(\frac{\theta_0}{2} \right) + \frac{1^2 3^2 5^2}{2^2 4^2 6^2} \sin^6 \left(\frac{\theta_0}{2} \right) + \dots \right)$$

Truncating this series after these three terms gives a good approximation that works well up to quite large angles. (At $\theta_0 = 60^\circ$, it is accurate to less than 0.2%.) We will refer to this as the approximate large-angle formula.

$$T = \frac{2\pi}{\omega} \left(1 + \frac{1^2}{2^2} \sin^2 \left(\frac{\theta_0}{2} \right) + \frac{1^2 3^2}{2^2 4^2} \sin^4 \left(\frac{\theta_0}{2} \right) \right) \quad \text{(approximate large-angle formula)}$$

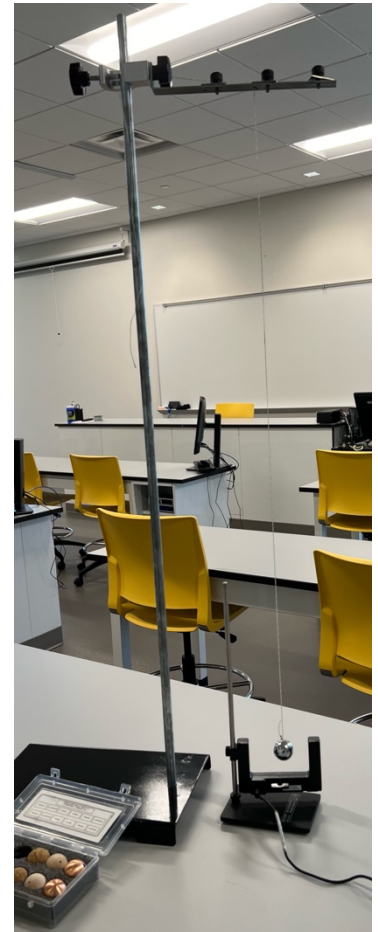
At any angle, the period of a pendulum is independent of the mass of the bob. We have neglected air resistance (friction) in this analysis. The force of air resistance on a bob depends on its size, shape and texture.

For a given size bob, friction has a larger effect on lighter bobs. When we want to minimize the effect of air resistance we will use the densest bob of lead or some other heavy metal.

Procedure

Setup

1. Set up the equipment as shown to the right. Place the support rod and base on the lab table and attach the pendulum clamp to near the top of the rod.
2. Cut a piece of string long enough to reach from the clamp to the table top. Tie the string to the lead bob, keeping in mind that you will need to swap out the bobs in part (B). Attach the string to the pendulum clamp. The clamp is designed to make it easy to vary the length of the string.
3. Place the photogate just above the table under the clamp with the open end pointing up. The swinging bob should break the beam of the photogate.
4. Turn on the Pasco interface at your lab station and plug the photogate into Digital Channel 1.
5. In the “Physics Folder” on your computer desktop open Pendulum.cap .
6. In *Capstone*, click **Record** and allow the pendulum bob to swing through the photogate. The interface is very simple; there is a column recording the periods. Note that the program records the time interval between every other time the bob breaks the beam, so it gives full periods. To end the recording, click the **Stop** button.
7. To vary the length of the pendulum, raise or lower the clamp and pull the string through the clamp. You should not need to change the position of the photogate.



Data Recording – Part (A)

1. In part (A) we will study the pendulum at large angles and test the approximate large angle formula. Use the lead or steel bob to minimize the effect of friction. Keep the length of the pendulum constant and as long as is practical for your setup. For theoretical calculations use the approximate large-angle formula.
2. Using a scale in the laboratory, measure the mass of the bob and record that value. Also, with a meter stick measure the length of the pendulum. This length is the distance from the fixed point of the string to the center of the bob. For that length, calculate the small-angle period and record that.
3. Using a protractor raise the string and bob to the appropriate amplitude angle θ_0 from vertical. Hit record and release. The column of values for the period will give many times that should all be quite close. As time goes on friction will cause the amplitude to decay, so take the first time; if the first recorded time is way off from the other times, there could be an issue with that value and choose the next reading. Record the value for the period as the experimental period T_{exp} in the data table and continue for all the different θ_0 values.

- To calculate the theoretical period T_{theo} , use the approximate large-angle period formula. This is a tedious and difficult calculation to repeat, so it is advised that you automate the process using either an Excel formula, a programmed formula in a calculator or using Mathematica.
- Calculate two different percent errors for each angle: the percent error between T_{exp} and T_{theo} , and the percent error between T_0 and T_{theo} . For both percent errors take the theoretical value T_{theo} as the accepted value.

Data Recording – Part (B)

- In part (B) we will study the effect of mass on the period. Neglecting friction, the period of a pendulum is independent of mass. The only dependence is due to friction effects. You should notice that the very light bobs are slowed down somewhat. Use four different bobs: lead or steel, aluminum (or some other metal), wood and cork. It is not necessary to measure the angle but keep it small. It is essential to keep this length constant as you swap out the different bobs. Use a long string as in part (A). Use the small-angle formula for the theoretical calculations: $T_{\text{theo}} = T_0$.
- Using a scale measure the masses of the bobs and record those values. Also, measure and record the length of the pendulum.
- For each mass, calculate the percent error between T_{exp} and T_{theo} , taking the theoretical value $T_{\text{theo}} = T_0$ as the accepted value.

Data Recording – Part (C)

- In part (C) we will study the effect of length on the small-angle period. It is not necessary to measure the angle but as in part (B) keep it small. Use the lead or steel bob as in part (A) and record its mass.
- You want five different lengths that are well spread-out. If L_0 is the length from the previous parts then choose values that are approximately L_0 , $0.8L_0$, $0.6L_0$, $0.4L_0$ and $0.2L_0$. Use the small-angle formula for the theoretical calculations: $T_{\text{theo}} = T_0$.
- For each length, calculate the percent error between T_{exp} and T_{theo} , taking the theoretical value $T_{\text{theo}} = T_0$ as the accepted value.
- You will be asked to plot a graph of T_{exp}^2 vs. L . From the slope of that graph, you will find an experimental value of the acceleration due to gravity g .

Appendix – Derivation of the Series Formula

For those who are interested, the derivation of the series solution for the period is included here. It is a bit ugly, but it is a nice application of the material you learn in Calculus II. We begin with an expression for the energy of a pendulum. The kinetic energy K and the potential energy U can be written as:

$$K = \frac{1}{2} I \omega^2 \text{ where } I = mL^2 \text{ and } U = mgL(1 - \cos \theta)$$

We can write the total energy in terms of the angle amplitude θ_0 . The expression for the conservation of mechanical energy becomes.

$$E = \frac{1}{2}mL^2 \left(\frac{d\theta}{dt}\right)^2 + mgd(1 - \cos\theta) = mgd(1 - \cos\theta_0)$$

With a definition of ω and a half-angle formula.

$$\omega = \sqrt{\frac{mgd}{I}} \quad \text{and} \quad 1 - \cos\theta = 2 \sin^2 \frac{\theta}{2}$$

we can rewrite the energy expression:

$$\left(\frac{d\theta}{dt}\right)^2 = 4\omega^2 \left(\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right)$$

and from this solve for dt .

$$dt = \frac{d\theta}{2\omega \sqrt{\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}}$$

It takes a quarter of a period $T/4$ for θ to vary from 0 to $\pi/2$. This allows us to write an integral expression for the period.

$$T = 4 \int_0^{\pi/2} \frac{d\theta}{2\omega \sqrt{\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)}}$$

Define the parameter $\lambda = \sin(\theta_0/2)$ and make the change of variables $x = \frac{\sin(\theta/2)}{\sin(\theta_0/2)}$. This gives

$$dx = \frac{1}{2\lambda} \cos\left(\frac{\theta}{2}\right) d\theta \Rightarrow d\theta = \frac{2\lambda dx}{\cos\left(\frac{\theta}{2}\right)} = \frac{2\lambda dx}{\sqrt{1 - \sin^2\left(\frac{\theta}{2}\right)}} = \frac{2\lambda dx}{\sqrt{1 - \lambda^2 x^2}}$$

To interpret the variable x , the fraction of the maximum potential energy is x^2 . Completing the change of variables, the expression in the denominator can be rewritten as well

$$\sqrt{\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)} = \lambda\sqrt{1 - x^2}$$

and the limits of integration become 0 and 1.

$$T = \frac{4}{\omega} \int_0^1 \frac{dx}{\sqrt{1 - \lambda^2 x^2} \sqrt{1 - x^2}}$$

This integral cannot be solved in closed-form, meaning that we cannot write the result in terms of standard functions. Because of this, we will write it as a series. Before doing that, it should be pointed out that in applied math there are a large number of “special functions” specifically defined as solutions to strange integrals. This period integral turns out to be something known as an “elliptic integral” K , where $T = \frac{4}{\omega} K(\sin^2(\theta_0/2))$.

To evaluate the Taylor (Maclaurin) expansion of $(1 - \lambda^2 x^2)^{-1/2}$, use the generalization of the binomial expansion.

$$(1 - t)^q = 1 + qt + \frac{q(q - 1)}{2} t^2 + \frac{q(q - 1)(q - 2)}{3!} t^3 + \dots$$

This gives:

$$(1 - (\lambda x)^2)^{-1/2} = 1 + \frac{1}{2}(\lambda x)^2 + \frac{1}{2!} \frac{1 \cdot 3}{2} (\lambda x)^4 + \frac{1}{3!} \frac{1 \cdot 3 \cdot 5}{2 \cdot 2} (\lambda x)^6 + \dots$$

$$(1 - (\lambda x)^2)^{-1/2} = \sum_{p=0}^{\infty} \frac{(2p-1)!!}{2^p p!} (\lambda x)^{2p} = \sum_{p=0}^{\infty} \frac{(2p-1)!!}{(2p)!!} (\lambda x)^{2p}$$

Inserting this into the integral give an infinite sum over integrals.

$$T = \frac{4}{\omega} \sum_{p=0}^{\infty} \lambda^{2p} \frac{(2p-1)!!}{(2p)!!} \int_0^1 \frac{x^{2p} dx}{\sqrt{1-x^2}}$$

To evaluate these integrals, use the trig substitution: $x = \cos \theta$.

$$I_p = \int_0^1 \frac{x^{2p} dx}{\sqrt{1-x^2}} = - \int_{\pi/2}^0 \frac{\cos^{2p} \theta \sin \theta d\theta}{\sqrt{1-\cos^2 \theta}} = \int_0^{\pi/2} \cos^{2p} \theta d\theta$$

Clearly $I_0 = \pi/2$. To find the general I_p integrate by parts.

$$I_p = \int_0^{\pi/2} \cos^{2p-1} \theta d\sin \theta = (\sin \theta \cos^{2p-1} \theta) \Big|_0^{\pi/2} = - \int_0^{\pi/2} \sin \theta d\cos^{2p-1} \theta$$

$$I_p = (2p-1) \int_0^{\pi/2} \cos^{2p-2} \theta \sin^2 \theta d\theta = (2p-1) \int_0^{\pi/2} \cos^{2p-2} \theta (1 - \cos^2 \theta) d\theta = (2p-1)(I_{p-1} - I_p)$$

We can now write I_p in terms of I_{p-1} and then relate it to $I_0 = \pi/2$.

$$2pI_p = (2p-1)I_{p-1} \implies I_p = \frac{2p-1}{2p} I_{p-1} = \frac{(2p-1)!!}{(2p)!!} I_0 = \frac{\pi (2p-1)!!}{2 (2p)!!}$$

Putting this all together we get the result we want.

$$T = \frac{2\pi}{\omega} \sum_{p=0}^{\infty} a_p \sin^{2p} \left(\frac{\theta_0}{2} \right) \quad \text{where} \quad a_p = \left(\frac{(2p-1)!!}{(2p)!!} \right)^2$$

Pendulum – Data Sheet

Name _____ Group _____

(A) The Pendulum at Large Angles

Mass: $m =$ _____ Length: $L =$ _____ Small Angle Period: $T_0 =$ _____

Amp. θ_0	Period		% Error (between T_{exp} and T_{theo})	% Error (between T_0 and T_{theo})
	Experimental: T_{exp}	Theoretical: T_{theo}		
5°				
10°				
20°				
30°				
45°				
60°				

(B) The Effect of the Mass of the Bob

Length: $L =$ _____ Use a small angle.

Mass: m	Period		% Error
	Experimental: T_{exp}	Theoretical: T_{theo}	

(C) The Effect of Length and a Calculation of g

Mass: $m =$ _____

Use a small angle.

Length: L	Period		% Error
	Experimental: T_{exp}	Theoretical: T_{theo}	

Question 1 Using the data from Part (C), graph $(T_{\text{exp}})^2$ versus L and include the best-fit line and the equation of the best-fit line. Find the slope of the best-fit line. Attach the graph to this data sheet.

slope = _____

Question 2 Starting with $T = 2\pi \sqrt{L/g}$, find the theoretical expression for the slope of a T^2 versus L graph.

The Period of the Simple Pendulum

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Question 3 Using the expression found in Question 2 solve for g_{exp} , the experimental value of g , from your experimental value of the slope in Question 1. Compare the experimental value of g to the accepted value of 9.80 m/s^2 .

$g_{\text{exp}} =$ _____ % Error in $g =$ _____

Question 4 On the moon gravity is approximately $1/6$ as strong as on earth. For pendulums of the same length find the ratio of the period on the moon to the period on the earth, $T_{\text{moon}}/T_{\text{earth}}$?

Question 5 If the force of air resistance on each bob is approximately the same on all the bobs used, then explain why in part (B), the lighter bobs are affected by it more.

Boyle's Law

Equipment and Setup: Syringe, Pressure sensor, Temperature sensor, Capstone file – Boyle's Law.cap

Part I. Theory

When temperature is not very low and pressure is not very high, gases follow ideal gas law:

$$pV = NkT, \quad \text{or} \quad pV = nRT$$

When temperature T is kept constant, the product of pressure p and volume V for a certain amount of gas is constant, which is called Boyle's law, discovered by British scientist Robert Boyle in 1662 by experiment. That is: $p_1V_1 = p_2V_2 = p_3V_3 = \dots$. The pV diagram is like figure 1.

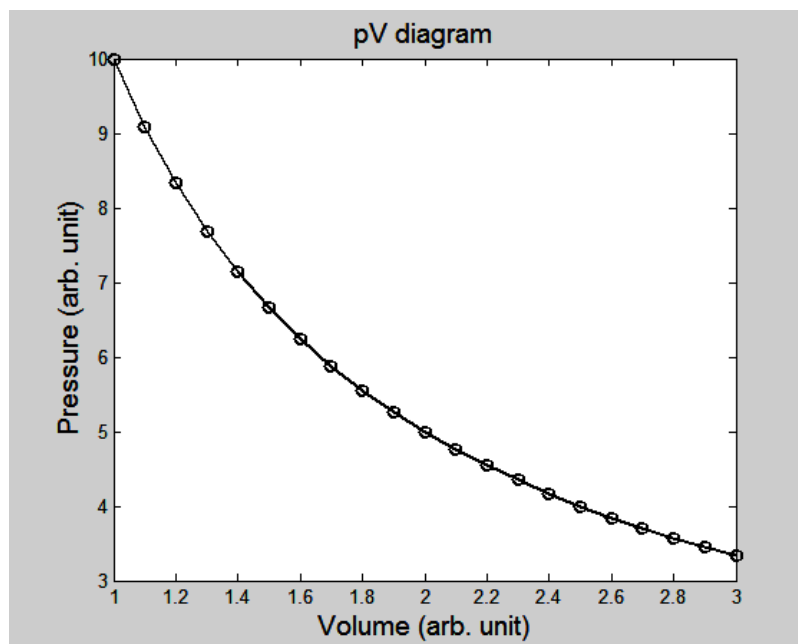


Figure 1. The pV diagram at constant temperature.

The purpose of this experiment is to verify this law.

Part II. Procedure

1. Set the plunger for a volume of 50 cc (mL). Connect the syringe to pressure sensor unit and the temperature sensor unit; then connect the pressure sensor unit to port A and the temperature sensor unit to port B on Capstone box, as shown in Fig. 2.

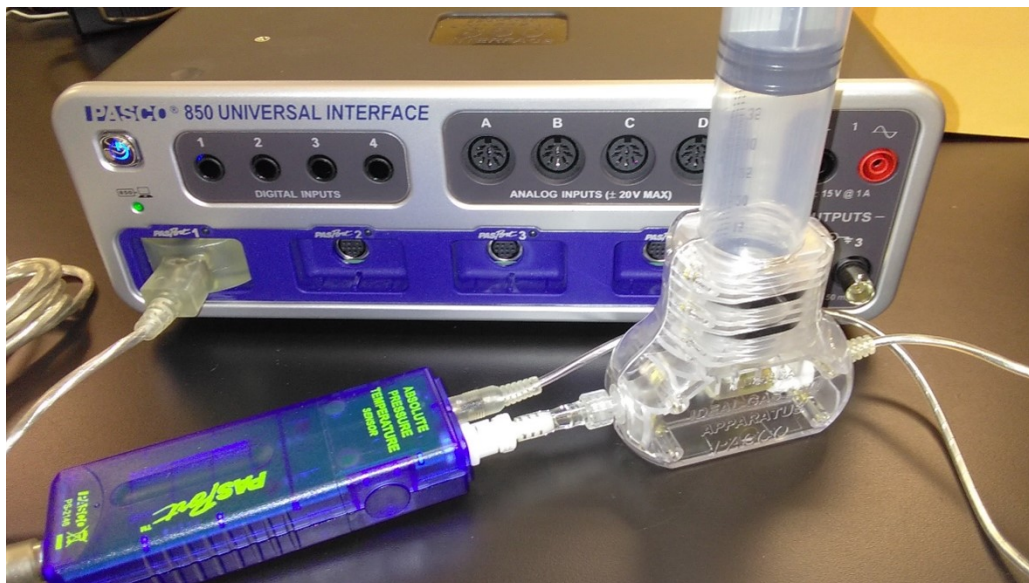


Fig. 2. (a) Pressure sensor and temperature sensor connected to syringe.
(b) Sensors connected to Capstone box.

2. Double click on Boyle's Law in Capstone in Physics folder.
3. Click Constant Temp on the menu.
4. Choose Data Summary if not chosen yet; the screen should be like fig. 3.

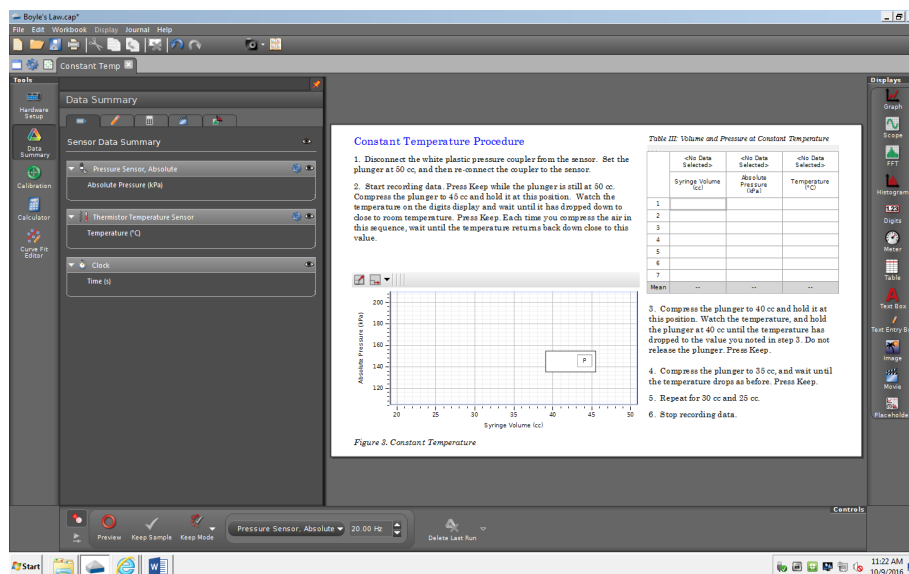


Fig. 3. Screenshot of Boyles' law lab

- Click Preview to start recording data. Type "50" in the box labeled Volume in the first row of Table 1 (It means that the volume in the syringe is 50 cubic centimeters, or 50 mL). Click Keep Sample while the plunger is still at 50 cc.
- Now push the plunger of the syringe to 45 mL and type "45" in the box of Volume in the 2nd row of table. Wait for 30 seconds (this is about the time needed for temperature to get back to room temperature again).
- Click Keep Sample while the plunger is still at 45 cc. Compress the plunger to 40 cc and hold it at this position for 30 seconds.
- Continue this process to get data for volumes of 35 cc, 30 cc, 25 cc, and 20 cc. You should see a graph like Fig. 1 in this manual.
- Record the data to Table I and find pV values. (Note: pay attention to the units of pressure and volume to get correct unit of Joule for their product)
- Disconnect, YES, DISCONNECT, the syringe to pressure sensor unit first, and change the volume to 20 cc. Do 2nd trial to fill the Table 2, starting with 20 cc this time.

Part III. Data Tables

Table 1 (Trial 1)		
V (mL)	P (kPa)	pV (J)
50		
45		
40		
35		
30		
25		
20		
Average $pV =$		J

Table 2 (Trial 2)		
V (mL)	P (kPa)	pV (J)
20		
25		
30		
35		
40		
45		
50		
Average $pV =$		J

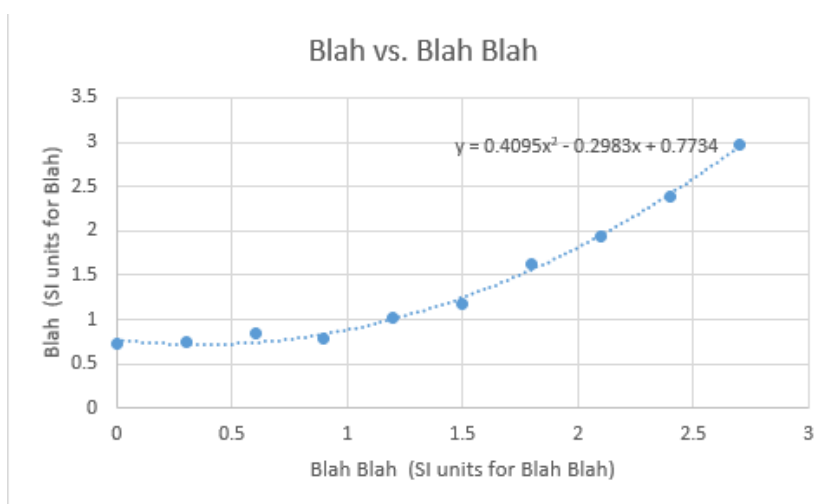
Part IV. Problems

1. Plot the pV diagrams (graphs of p vs. V) for both trials, which should be turned in with the lab report.
2. Assume room temperature is $23\text{ }^{\circ}\text{C} = 296\text{ K}$, find the number of moles and number of molecules of air in the syringe by using the average values of pV for each of the two trials. Show your work and both answers below.
3. Suppose that at room temperature and around 100 kPa (which is about the temperature and pressure when you start the trial 1), the air mass density is 1.2754 kg/m^3 . Find the mass in kg and weight in Newtons of air inside the syringe for trial 1.

Appendix - Notes on Graphing

General Comments

- All graphs must be created on a computer using Excel or some other graphing software.
- The graph must be properly labeled.
 - At the top of each graph there should be a title describing the graph; this should include the variables that are plotted, for example a distance versus time graph could be labeled as “distance vs. time” or just as “ x vs. t ”. It should also distinguish the graph from the other graphs in your experiment’s write-up; for example, if there are multiple x vs. t plots then the title should read something like: “ x vs. t for Part (A)”
 - Each axis must be labeled, and the units should be included in brackets. For the example of a distance x use the label: “ x (m)”.
- When plotting y vs. x , the horizontal axis is x . Always think y vs. x .
- All graphs must have a uniform scale along both axes. The axes should cross at the point (0,0).
- For any graph you must plot the **data points**, the **best-fit curve** and the **equation of the best-fit curve**.



Graphing with Excel

- When using Excel always use the scatter plot format. Other formats will likely cause problems with maintaining a uniform scale.
- Put the data in columns; if it is a graph of y vs. x , then the x column is on the left and y column is on the right. Highlight the data and then click the **Insert** tab. Then **Chart** > **Scatter** then choose the option “Scatter with only markers”.
- There is a **Chart Design** menu item at the top that will return you to the chart ribbon.
- Select the **Add Chart Element** tab (at the top left of the ribbon) to add the labels. To label Axes click **Axes Titles**. There you will find **Primary Horizontal** and **Primary Vertical**, click on one then then the next and appropriately label each axis. Labeling the graph is done with the **Chart Title** choice or you can just click on the placeholder title and edit that.
- If the axis origin is not through (0,0) you can force it. Under the **Add Chart Element** tab click **Axes**, and then **More Axis Options**. This opens a **Format Axis** toolbar at the right. When there under Vertical axis crosses, uncheck **Automatic** and check **Axis value** and add (0,0).

- If you want to plot two data sets on the same graph right-click **Select Data** and then click to add a new set.
- Now you must add the best-fit line or trendline. Once again, go to the **Add Chart Element** tab and click **Trendline > More Trendline Options**. That opens a **Format Trendline** toolbar at the right. Under **Trendline Options** choose **Linear** for the best-fit line or choose **Polynomial** then **Order 2** for the best-fit parabola. At the bottom of that toolbar always check **Display Equation on chart**. Sometimes the equation given for the trendline lacks significant figures. For instance, suppose one of the parameters in the trendline is 0.000003, although there are seven digits displayed, there is only one significant figure. To get more digits displayed then right-click on the trendline and choose **Format Trendline Label**, then choose an appropriate number of decimal places to get enough significant digits.
- Remember always that a graph of something versus something else, the first thing is the y axis and the second is the x . It is always y vs. x .
- If you mistakenly reverse the data when plotting you can swap the x and y values instead of starting the graph from scratch. To the right on the **Chart Design** ribbon choose **Select Data**. There you can manually change the references for the x and y values.