

Chapter L - Problems

Blinn College - Physics 2325 - Terry Honan

Problem L.1

Suppose that Young's modulus for bone is $1.6 \times 10^{10} \text{ N/m}^2$ and the maximum stress a bone can handle is $1.4 \times 10^8 \text{ N/m}^2$ without breaking. Assume a femur to be a right cylinder of diameter 2.84 cm and of length 48 cm.

- (a) What is the maximum force the femur could handle?
(b) With the force from part (a), how much does a 48 cm length of bone compress?

Solution to L.1

- (a) The stress is the force per area.

$$1.4 \times 10^8 = \frac{F_{\max}}{A} = \frac{F_{\max}}{\pi(0.0284/2)^2} \Rightarrow F_{\max} = 88\,700 \text{ N}$$

- (b) The definition of Young's modulus gives our answer.

$$Y = \frac{F/A}{\Delta L/L_0} \Rightarrow \Delta L = \frac{F/A}{Y} L_0 = \frac{1.4 \times 10^8}{1.6 \times 10^{10}} 0.48 = 4.2 \text{ mm}$$

Problem L.2

A 300 kg mass hung from a steel wire stretches it by 0.2%. What is the diameter of the wire?

Solution to L.2

For steel: $Y = 20 \times 10^{10} \text{ N/m}^2$. Stretching by 0.2% implies that $\Delta L/L_0 = 0.002$.

$$Y = \frac{F/A}{\Delta L/L_0} \Rightarrow \pi \left(\frac{d}{2}\right)^2 = A = \frac{F}{Y \Delta L/L_0} = \frac{300 \times 9.8}{20 \times 10^{10} \times 0.002} = 7.35 \times 10^{-6}$$
$$\Rightarrow d = 2 \sqrt{\frac{A}{\pi}} = 3.05 \text{ mm}$$

Problem L.3

The maximum shear stress that steel can withstand is $4.0 \times 10^8 \text{ N/m}^2$.

- (a) What shear force is needed break a 0.5 cm diameter bolt?
(b) What shear force is needed to punch a 1 cm diameter hole in a 1 mm thick steel sheet?
(c) What maximum shear strain $\Delta x/h$ in steel is associated with this maximum shear stress?

Solution to L.3

- (a) The relevant area is the cross-section of the bolt.

$$4 \times 10^8 = \frac{F_{\max}}{A} = \frac{F_{\max}}{\pi 0.0025^2} \Rightarrow F_{\max} = 7850 \text{ N}$$

- (b) The area the shear force acts on is the circumference times the thickness, $A = \pi d \times t$.

$$4 \times 10^8 = \frac{F_{\max}}{2\pi r t} = \frac{F_{\max}}{\pi 0.01 \times 0.001} \Rightarrow F_{\max} = 12\,600 \text{ N}$$

- (c) The shear modulus for steel is $S = 8.4 \times 10^{10} \text{ N/m}^2$. Write the maximum strain as $\Delta x_{\max}/h$

$$8.4 \times 10^{10} = S = \frac{F_{\max}/A}{\Delta x_{\max}/h} = \frac{4 \times 10^8}{\Delta x_{\max}/h} \Rightarrow \frac{\Delta x_{\max}}{h} = 0.00476$$

Problem L.4

Consider a pulse that in SI units has the shape

$$u = f(x) = \frac{8}{x^2+4}.$$

Write this as a function $u(x, t)$ that describes this pulse moving in the positive x direction with a speed of 3 m/s .

Solution to L.4

A pulse of shape $u = f(x)$ moving in the positive x -direction with speed v takes the form: $u = f(x - vt)$. Using this form of $f(x)$ with $v = 3 \text{ m/s}$ gives:

$$f(x) = \frac{8}{x^2+4} \implies u = \frac{8}{(x-3t)^2+4}.$$

Problem L.5

What are the speed and direction of a pulse on a string that (in SI units) has the form:

$$y(x, t) = 0.04 e^{-\left(\frac{x+0.03t}{0.06}\right)^2}.$$

Solution to L.5

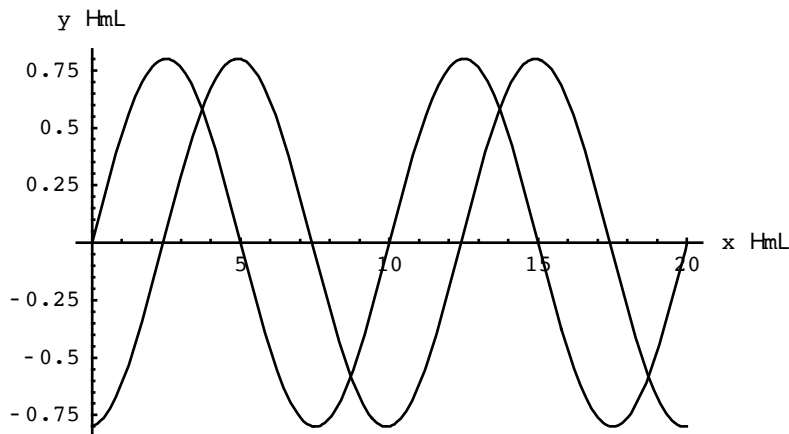
A wave of the form: $y(x, t) = f(x \mp vt)$ represents a pulse of shape $y = f(x)$ moving in the $\pm x$ direction with speed v . Here we have $y(x, t) = 0.04 e^{-(x/0.06)^2}$ with $v = 0.03 \text{ m/s}$ moving in the negative x direction.

Problem L.6

A sinusoidal pulse on a string has the mathematical form $y(x, t) = (0.80 \text{ m}) \sin\left[\frac{2\pi}{10}(x - 4t)\right]$. Plot the y vs. x graph at $t = 0 \text{ s}$. By the time $t = 0.6 \text{ s}$ how much has the pulse shifted. On the same graph plot y vs. x at $t = 0.6 \text{ s}$.

Solution to L.6

After $t = 0.6 \text{ s}$ the graph has shifted by $vt = 4 \times 0.6 = 2.4 \text{ m}$.



Problem L.7

A string with a linear density of $\mu = 4 \times 10^{-3} \text{ kg/m}$ is given a tension of 360 N . What is the speed of waves on this string?

Solution to L.7

The speed of waves on a stretched string is $v = \sqrt{T/\mu}$ where T is the tension in the string and μ is the linear density (mass/length) of the string. Here, $T = 360 \text{ N}$ and $\mu = 4 \times 10^{-3} \text{ kg/m}$ giving $v = 300 \frac{\text{m}}{\text{s}}$.

Problem L.8

The elastic limit for steel is $S_{\max} = 2.7 \times 10^9 \text{ N/m}^2$; this is the maximum stress (force per area) that steel under tension can withstand. S_{\max} is the largest value that T/A , the tension per area, can have without a wire breaking. If the density of steel is 7860 kg/m^3 then what is the largest

speed a wave can travel down a steel wire?

Solution to L.8

The linear density μ (mass/length) is related to the volume density ρ (mass/volume) by $\lambda = \rho A$, where A is the cross-sectional area of the wire. This is easy to show: The volume of a wire of length L and area A is LA . Mass = $\rho \times \text{Volume} = \rho LA$ and $\lambda = \text{Mass}/L = \rho A$.

The maximum stress Stress_{\max} gives the maximum tension: $T_{\max} = S_{\max} \times A$, which then gives the maximum wave speed.

$$v_{\max} = \sqrt{\frac{T_{\max}}{\mu}} = \sqrt{\frac{S_{\max} A}{\rho A}} = \sqrt{\frac{S_{\max}}{\rho}} = \sqrt{\frac{2.7 \times 10^9}{7.86 \times 10^3}} = 586 \frac{\text{m}}{\text{s}}$$

Problem L.9

A 30 m long copper wire with a 1.2 mm diameter is stretched to a tension of 200 N. How long does it take for a pulse to travel the length of the wire? The density of copper is $\rho = 8.92 \times 10^3 \text{ kg/m}^3$.

Solution to L.9

The cross-sectional area of the wire is

$$A = \pi r^2 = \pi \times 0.0006^2 = 1.1310 \times 10^{-6} \text{ m}^2.$$

We saw in the previous problem that the linear density μ of a wire with density ρ and cross-sectional area A is $\mu = \rho A$. The speed of a wave on a wire with tension T , cross-section A and density ρ is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{200}{8.92 \times 10^3 A}} = 141 \frac{\text{m}}{\text{s}}.$$

To get the time from the distance d and speed simply use $d = vt$ giving:

$$t = \frac{d}{v} = \frac{30}{141} = 0.213 \text{ s}.$$

Problem L.10

A sinusoidal wave on a string has the form

$$y(x) = (15 \text{ cm}) \cos\left[\left(\frac{\pi}{20} \text{ cm}^{-1}\right)x - (16\pi \text{ s}^{-1})t\right].$$

- Plot the motion of the position $x = 0$ as a function of time and find its period and frequency.
- What is the maximum speed of this point ($x = 0$) on the string?
- What are the wavelength and speed of this wave?

Solution to L.10

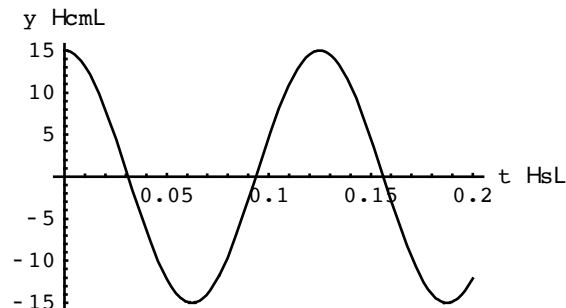
(a) At $x = 0$ we have

$$y(x) = A \cos(\omega t), \text{ where } A = 15 \text{ cm and } \omega = 16\pi \text{ s}^{-1}.$$

The period T and frequency f are

$$T = \frac{2\pi}{\omega} = \frac{1}{8} = 0.125 \text{ s and } f = \frac{\omega}{2\pi} = \frac{1}{T} = 8 \text{ Hz}.$$

With this we can plot the function.



(b) This motion is an example of simple harmonic motion, where the maximum speed is given by

$$v_{\max} = \omega A = 16\pi \times 15 = 240\pi = 754 \frac{\text{cm}}{\text{s}}.$$

(c) The wave number k of this wave gives the wavelength.

$$k = \frac{\pi}{20} \text{ cm}^{-1} \implies \lambda = \frac{2\pi}{k} = 40 \text{ cm}$$

The speed can be found by using $v = f\lambda$ or directly in term of what is giving by using

$$v = \frac{\omega}{k} = \frac{16\pi}{\pi/20} = 320 \frac{\text{m}}{\text{s}}.$$

Problem L.11

As a sinusoidal wave passes, a point on a string makes 50 complete vibrations in 20 s. In the same time a crest (maximum) of the wave moves a distance of 4 m. What is the frequency, speed and wavelength of this wave?

Solution to L.11

50 vibrations in 20s gives a frequency of

$$f = \frac{50}{20} = 2.5 \text{ Hz}.$$

4 m in 20 s gives the speed.

$$v = \frac{4}{20} = 0.2 \frac{\text{m}}{\text{s}}$$

We can now find the wavelength.

$$f\lambda = v \implies \lambda = \frac{v}{f} = \frac{0.2}{2.5} = 0.08 \text{ m}$$

Problem L.12

A 15 m length of rope has a mass of 0.6 kg and is given a tension of 500 N. What power is required to put a wave with an amplitude of 20 cm and a frequency of 3 Hz?

Solution to L.12

The linear density is $\mu = M/L = 0.6/15 = 0.04 \text{ kg/m}$. The speed of the wave is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{500}{0.04}} = 111.80 \text{ m/s}$$

The angular frequency follows from the frequency.

$$\omega = 2\pi f = 2\pi \cdot 3 = 18.850 \text{ s}^{-1}$$

Using $A = 0.20 \text{ m}$ we can now find the power transmitted down a string.

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} \cdot 0.04 \times 1.8850^2 (0.20)^2 \cdot 111.80 = 31.8 \text{ W}.$$

Problem L.13

A wave of the form

$$y(x, t) = (0.12 \text{ m}) \sin[(0.8 \text{ m}^{-1})x + (40 \text{ s}^{-1})t]$$

travels down a string with a linear density of 8 g/m .

- What is the speed of the wave and in what direction is it moving?
- What are the wavelength and frequency of this wave?
- What is the tension in the string?
- What is the power transmitted by this wave?

Solution to L.13

Since the general form is $y(x, t) = A \sin(kx \mp \omega t - \phi)$, we can conclude that: $A = 0.12 \text{ m}$, $k = 0.8 \text{ m}^{-1}$ and $\omega = 40 \text{ s}^{-1}$. We are also

given that $\mu = 0.008 \frac{\text{kg}}{\text{m}}$.

(a) The positive sign before ω implies it is moving in the negative x direction. The speed is

$$v = \frac{\omega}{k} = \frac{40}{0.8} = 50 \frac{\text{m}}{\text{s}}$$

(b) The wavelength and frequency come from the wave number and angular frequency.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.8} = 7.85 \text{ m} \quad \text{and} \quad f = \frac{\omega}{2\pi} = \frac{40}{2\pi} = 6.37 \text{ Hz}$$

(c) The speed and linear density give the tension.

$$v = \sqrt{\frac{T}{\mu}} \implies T = v^2 \mu = 50^2 \times 0.008 = 20 \text{ N}$$

(d) The power transmitted is

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} \times 0.008 \times 40^2 (0.12)^2 50 = 4.61 \text{ W}.$$

Problem L.14

The intensity of sunlight varies with the distance from the Sun. At the Earth's distance the intensity is 1390 W/m^2 .

- (a) What is the intensity of sunlight at Neptune:
 (b) What is the rate that the Sun radiates energy in Watts?

Solution to L.14

The intensity I of a three-dimensional wave is related to the average power $\bar{\mathcal{P}}$ by $I = \bar{\mathcal{P}}/A$. Since the light that leaves the sun at some instant is at a later time distributed over a sphere the relevant area is that of a sphere $A = 4\pi r^2$.

$$I = \frac{\bar{\mathcal{P}}}{4\pi r^2}$$

Here $\bar{\mathcal{P}}$ is the total power radiated by the sun and r is the distance from the sun.

(a) Since $\bar{\mathcal{P}}$ is the same for any r we may get the following ratio:

$$\frac{I_2}{I_1} = \frac{1}{(r_2/r_1)^2}$$

The distance of the Earth from the Sun is $1.50 \times 10^{11} \text{ m}$ and the distance from the Sun to Neptune is $4.50 \times 10^{12} \text{ m}$.

$$I_2 = \frac{I_1}{(r_2/r_1)^2} = \frac{I_2}{(45/1.5)^2} = \frac{I_2}{900} = \frac{1390}{900} = 1.54 \frac{\text{W}}{\text{m}^2}$$