Chapter M - Problems

Blinn College - Physics 2325 - Terry Honan

Problem M.1

A train moving at 60 m/s blows its horn at a pitch of 450 Hz. A car drives along a road parallel to the train tracks moving in the same direction at 25 m/s. What is the frequency heard by the car as the train approaches and as the train moves away? There are two answers.

Solution to M.1

The frequency heard by the observer f' can be written in terms of the frequency of the source f

$$f' = \frac{v + v_o}{v - v_s} f$$

where the velocity of the source and observer are v_s and v_o respectively and where both v_s and $v_o$are positive when directed toward the other. Tee wave speed is v.

$$f = 450\text{ Hz} \quad \text{and} \quad v = 344\,\frac{\text{m}}{\text{s}}$$

As the train approaches we have $v_s = 60$ m/s but since the car is moving in the same direction it is moving away and $v_o = -25$ m/s.

$$f' = \frac{v + v_o}{v - v_s} f = 505\text{ Hz}$$

As the train moves away we have $v_s = -60$ m/s but now the car is moving toward the train, so $v_o = 25$ m/s.

$$f' = \frac{v + v_o}{v - v_s} f = 411\text{ Hz}$$

Problem M.2

A guitar string of length 0.72 m and a linear density of 0.18 grams/m.
(a) What tension must be used to tune this string to a fundamental frequency of 220 Hz?
(b) What are the next three lowest frequencies (after the fundamental) produced by this string?
(c) Moving a finger along the frets of a guitar decreases the effective length of the string while keeping the tension and thus wave speed the same. Where must ones finger be placed to make this string oscillate with a fundamental frequency of 280 Hz?

Solution to M.2

(a) Here we have $L = 0.72$ m and the fundamental frequency of $f_1 = 220$ Hz. The fundamental frequency can be found in terms of the wave speed and string length.

$$f_1 = \frac{v}{2\,L}$$

This gives the wave speed.

$$v = f_1\,2\,L = 316.8\,\frac{\text{m}}{\text{s}}$$

The linear density of the string is $\mu = 0.18 \times 10^{-3}$ kg/m. Using the formula for the speed of waves on a string we can find the tension T.

$$v = \sqrt{\frac{T}{\mu}} \quad \Longrightarrow \quad T = \mu\,v^2 = 18.07\text{ N}$$

(b) The higher harmonic frequencies are integer multiples of the fundamental.

$$f_m = m\,f_1 \qquad (m = 1, 2, 3, \ldots)$$

It follows that the next three lowest are:

$$f_2 = 2\, f_1 = 440\ \text{Hz}\,, \quad f_3 = 3\, f_1 = 660\ \text{Hz}\,, \quad f_4 = 4\, f_1 = 880\ \text{Hz}$$

(c) Changing the length keeping the tension and speed fixed allows us to write the new fundamental frequency in terms of the old using a simple ratio where L is the original length and L' is the new length.

$$f_1 = \frac{v}{2\,L} \implies \frac{f_1'}{f_1} = \frac{1}{L'/L}$$

Now we can solve for the new length using $f_1 = 220$ Hz, $f_1' = 280$ Hz and $L = 0.72$ m.

$$\frac{f_1'}{f_1} = \frac{1}{L'/L} \implies L' = \frac{L}{f_1'/f_1} = 0.566\ \text{m}$$

Problem M.3

What is the length of an organ pipe with both ends open needed to produce a fundamental frequency of 220 Hz? Take the speed of sound to be 344 m/s. What are the next three lowest frequencies (after the fundamental) produced by this pipe?

Solution to M.3

We have $v = 344$ m/s and the fundamental frequency is $f_1 = 220$ Hz. In terms of the speed of sound and the pipe length is L we have

$$f_1 = \frac{v}{2\,L}.$$

Solving for L we get.

$$L = \frac{v}{2\,f_1} = 0.782\ \text{m}$$

The higher harmonic frequencies are integer multiples of the fundamental.

$$f_m = m\, f_1 \quad (m = 1, 2, 3, \ldots)$$

It follows that the next three lowest are:

$$f_2 = 2\, f_1 = 440\ \text{Hz}\,, \quad f_3 = 3\, f_1 = 660\ \text{Hz}\,, \quad f_4 = 4\, f_1 = 880\ \text{Hz}$$

Problem M.4

Now consider an organ pipe with one end opened and one end closed. What is the length of this pipe needed to produce a fundamental frequency of 220 Hz? Take the speed of sound to be 344 m/s. What are the next three lowest frequencies (after the fundamental) produced by this pipe?

Solution to M.4

Again we have $v = 344$ m/s and the fundamental frequency is $f_1 = 220$ Hz. The fundamental frequency in this case is

$$f_1 = \frac{v}{4\,L}.$$

Solving for L we get.

$$L = \frac{v}{4\,f_1} = 0.391\ \text{m}$$

The higher harmonic frequencies are integer multiples of the fundamental.

$$f_m = m\, f_1 \quad (m = 1, 3, 5, \ldots)$$

It follows that the next three lowest are:

$$f_3 = 3\, f_1 = 660\ \text{Hz}\,, \quad f_5 = 5\, f_1 = 1100\ \text{Hz}\,, \quad f_7 = 7\, f_1 = 1540\ \text{Hz}$$

Problem M.5

The note Middle C has a frequency of 262 Hz. A piano is tuned to Middle C using an accurate tuning fork. When the sound from the mistuned string is combined with the tuning fork a beat frequency of 12 Hz is heard. What possible frequencies can the mistuned string have?

Solution to M.5

For two frequencies f_1 and f_2 we have the beat frequency of $f_{\text{beat}} = |f_1 - f_2|$ where we have $f_{\text{beat}} = 12\,\text{Hz}$. We will take $f_2 = 262\,\text{Hz}$. (It doesn't matter which is f_1 and which is f_2.) Because of the absolut value we have two possible answers.

$$f_{\text{beat}} = |f_1 - f_2| \implies f_1 = f_2 \pm f_{\text{beat}} \implies f_1 = 250\,\text{Hz} \text{ and } f_1 = 274\,\text{Hz}$$