Physics 2326 - Formula List

- **Integrals**

$\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$ $\quad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$

$\int \frac{du}{u} = \ln|u| + C$ $\quad \int \frac{du}{\sqrt{u^2+a^2}} = \ln\left(u + \sqrt{u^2+a^2}\right) + C$

$\int e^{a u}\,du = \frac{1}{a} e^{a u} + C$ $\quad \int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

$\int \sin(a u)\,du = -\frac{1}{a}\cos(a u) + C$ $\quad \int \frac{du}{(u^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{u}{\sqrt{u^2+a^2}} + C$

$\int \cos(a u)\,du = \frac{1}{a}\sin(a u) + C$ $\quad \int \frac{u\,du}{(u^2+a^2)^{3/2}} = -\frac{1}{\sqrt{u^2+a^2}} + C$

- **Formulas from Geometry**

Sphere: Volume $= \frac{4}{3}\pi r^3$, Area $= 4\pi r^2$

Circle: Area $= \pi r^2$, Circumference $= 2\pi r$

Right Cylinder: Volume = Area $\times$ height

- **Coulomb's Law** $F = k_e \frac{|Q_1|\cdot|Q_2|}{r^2}$ (magnitude of force)

$\vec{F}_{21} = k_e Q_1 Q_2 \frac{\hat{r}_{12}}{r_{12}^2}$ where $\frac{\hat{r}_{12}}{r_{12}^2} = \frac{\vec{r}_{12}}{r_{12}^3}$

and where $\vec{r}_{12}$ is the vector from Q_1 to Q_2.
Charge quantization: $Q = n e$, n is an integer.

- **Electric Field** $\vec{E} = \vec{F}/q_0$ or $\vec{E} = \lim_{q_0\to 0} \vec{F}/q_0$, $\vec{F} = Q\vec{E}$ (force on Q)

Point Charge: $\vec{E} = k_e Q \frac{\hat{r}}{r^2} = k_e Q \frac{\vec{r}}{r^3}$, $E = k_e \frac{|Q|}{r^2}$

Discrete: $\vec{E} = k_e \sum_i Q_i \frac{\hat{r}_i}{r_i^2} = k_e \sum_i Q_i \frac{\vec{r}_i}{r_i^3}$

Continuous: $\vec{E} = k_e \int \frac{\hat{r}}{r^2}\,dq = k_e \int \frac{\vec{r}}{r^3}\,dq$

$\vec{E} = k_e Q \frac{z_0}{(R^2+z_0^2)^{3/2}}\hat{z}$ (z_0 from center of uniform ring)

- **Electric Flux** $\Phi = \int \vec{E}\cdot d\vec{A}$, uniform $\vec{E}$ and flat surface: $\Phi = \vec{E}\cdot\vec{A}$

Dot Product: $\vec{A}\cdot\vec{B} = A B\cos\theta = A_x B_x + A_y B_y + A_z B_z$

- **Gauss's Law** $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{\text{enclosed}}$
- **Potential and Potential Energy** $V = U/q_0$, $\Delta U = Q\,\Delta V$

For a charge distribution: $U = k_e \sum_{i<j} \frac{Q_i Q_j}{r_{ij}}$

Conservation of Energy: $K_i + U_i = K_f + U_f$, where $K = \frac{1}{2} m v^2$

- **Potential due to Charges** Point Charge: $V = k_e \frac{Q}{r}$

Discrete: $V = k_e \sum_i \frac{Q_i}{r_i}$, Continuous: $V = k_e \int \frac{dq}{r}$

- **Potential and Electric Field** $\Delta V = -\int \vec{E}\cdot d\vec{r}$, $\Delta V = -\vec{E}\cdot\Delta\vec{r}$ (uniform $\vec{E}$)

$E_x = -\frac{\partial V}{\partial x}$ and for y and z. also $E_r = -\frac{\partial V}{\partial r}$

- **Conductors in Electrostatics** At surface: $\vec{E} \perp$ surface and $E = \sigma/\varepsilon_0$

Inside: $\vec{E} = \vec{0}$, voltage is const., no excess charge.

- **Capacitance** $Q = C V$, C is the capacitance.

$C = \kappa C_0$, where C_0 = empty cap. and dielectric const. $= \kappa \geq 1$

$C_0 = \frac{\varepsilon_0 A}{d}$ ($\parallel$ plate), $C_0 = \frac{1}{k_e\left(\frac{1}{a}-\frac{1}{b}\right)}$ (sph.), $C_0 = \frac{2\pi\varepsilon_0 \ell}{\ln(b/a)}$ (cyl.)

- **Energy** $U = \frac{1}{2} C V^2 = \frac{Q^2}{2C} = \frac{1}{2} Q V$ (energy in a cap.)

$u = \frac{\varepsilon_0}{2} E^2 = \frac{\text{Energy}}{\text{Volume}}$ (energy density in a field)

- **Electric Dipoles** Dipole Moment: $\vec{p} = Q\vec{d}$, $\vec{d}$ from $-Q$ to Q

Torque: $\vec{\tau} = \vec{p}\times\vec{E}$, $\tau = p E\sin\theta$, Pot. Energy: $U = -\vec{p}\cdot\vec{E}$

- **Current and Current Density** $I = \frac{dQ}{dt}$ (current through surface)

$\vec{J}$ is the current density. $I = \int_{\text{surface}} \vec{J}\cdot d\vec{A}$, $I = J A$ (for a wire)

- **Drift Velocity** $I = n|q| v_d A$, $\vec{J} = n q \vec{v}_d$

$n = \frac{\text{\# of charge carriers}}{\text{vol.}}$, q = charge of charge carriers, $\vec{v}_d$ = drift vel.

- **Ohm's Law** $V = I R$, $R = \frac{\rho L}{A}$, $\vec{J} = \sigma\vec{E}$

σ = conductivity, $\rho = \frac{1}{\sigma}$ = resistivity

- **Temperature Dependence** $\Delta T = T - T_0$

$\Delta R = \alpha R_0\,\Delta T$, $R = R_0(1+\alpha\Delta T)$, $\Delta\rho = \alpha\rho_0\,\Delta T$, $\rho = \rho_0(1+\alpha\Delta T)$

- **Power** $\mathcal{P} = V I$, For a resistor: $\mathcal{P} = V I = I^2 R = \frac{V^2}{R}$
- **Combinations of Resistors**

Series: $R_{\text{eq}} = R_1 + R_2 + \ldots$, Parallel: $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}$

Node Reduction: $R'_{ij} = \frac{R_i R_j}{R_{\parallel}}$, where $R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

- **Combinations of Capacitors**

Series: $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)^{-1}$, Parallel: $C_{\text{eq}} = C_1 + C_2 + \ldots$

- **Kirchhoff's Rules** Junctions: $\sum I_{\text{in}} = \sum I_{\text{out}}$, Loops: $0 = \sum\Delta V$
- **Cross or Vector Product** $\vec{A}\times\vec{B} = \hat{u} A B\sin\theta$, right hand rule $\Rightarrow \hat{u}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

- **Magnetic Force on Particle** $\vec{F} = Q\vec{v}\times\vec{B}$

$\vec{v} \perp$ to uniform $\vec{B} \Longrightarrow$ circle with $r = \frac{m v}{|Q| B}$

- **Magnetic Force on Wire** $\vec{F} = I\int d\vec{r}\times\vec{B}$

$\vec{F} = I\vec{\ell}\times\vec{B}$ (straight segment, uniform field)

$V_{\text{Hall}} = v_d B L$ (Hall Voltage)

- **Magnetic Dipoles** Dipole Moment: $\vec{\mu} = N I\vec{A}$ (N is # of turns)

Torque: $\vec{\tau} = \vec{\mu}\times\vec{B}$, $\tau = \mu B\sin\theta$, Pot. Energy: $U = -\vec{\mu}\cdot\vec{B}$

- **Biot-Savart Law** $\vec{B} = \frac{\mu_0 I}{4\pi}\int d\vec{s}\times\frac{\hat{r}}{r^2}$

$\vec{B} = \hat{z}\frac{\mu_0 I}{4\pi R}\theta$ (at center of arc in xy-plane)

$\vec{B} = \hat{z}\frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$ (z from center of circle)

$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 - \sin\phi_1) = \frac{\mu_0 I}{4\pi a}\left(\frac{x_2}{\sqrt{x_2^2+a^2}} - \frac{x_1}{\sqrt{x_1^2+a^2}}\right)$ (segment)

$B = \frac{\mu_0 I}{2\pi r}$ (distance r from long wire)

- **Ampere's Law** $\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{\text{enclosed}}$

$B = \mu_0 n I$, $n = \frac{\text{\# of turns}}{\text{length}}$ (inside long solenoid)

- **Magnetic Flux** $\Phi_m = \int \vec{B}\cdot d\vec{A}$

For uniform field and flat surface: $\Phi = \vec{B}\cdot\vec{A}$

- **Faraday's Law** $\mathcal{E} = -N\,\frac{d\Phi}{dt}$, $\mathcal{E}_{ave} = -N\,\frac{\Delta\Phi}{\Delta t}$

AC Generator: $\Phi = B\,A\cos\omega t \implies \mathcal{E}(t) = N\,B\,A\,\omega\sin\omega t$

- **Motional EMF**

moving rod: $\mathcal{E} = B\,\ell\,v\ \left(\vec{B}\perp\vec{v}\perp\text{rod}\right)$

rotating rod: $\mathcal{E} = \frac{1}{2}B\,\ell^2\,\omega\ \left(\vec{B}\parallel\text{axis}\perp\text{rod}\right)$

- **Lenz's Law** (Choose + dir. and make a table of signs.)

Sign Φ is from dir. of field through loop.

$\frac{d\Phi}{dt}$ is same as (opposite to) sign of Φ when incr. (decr.).

$\Phi_{induced}$ sign is opposite to sign for $\frac{d\Phi}{dt}$.

Get direction of $\mathcal{E}$ or I by Right Hand Rule.

- **Maxwell's Equations**

$\oint\vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}Q_{enc}$, $\oint\vec{B}\cdot d\vec{A} = 0$

$\oint\vec{B}\cdot d\vec{r} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\Phi_e}{dt}$, $\oint\vec{E}\cdot d\vec{r} = -\frac{d\Phi_m}{dt}$

- **Inductance** Mutual: $\mathcal{E}_2 = -M\,\frac{dI_1}{dt}$, Self: $\mathcal{E} = -L\,\frac{dI}{dt}$

Long Solenoid: $L = \mu_0 n^2 A\,\ell = \mu_0\frac{N^2}{\ell}A$

Energy in Inductor: $U = \frac{1}{2}L\,I^2$

- **Energy Density** $u = \frac{1}{2\mu_0}B^2$ (in magnetic field)

$u = u_e + u_m = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$ (in electromagnetic field)

- **RC Circuits** $\tau = R\,C$ = time const., $\mathcal{E}$ = EMF

Discharging: $Q(t) = Q_0\,e^{-t/\tau}$ Charging: $Q(t) = C\,\mathcal{E}\left(1-e^{-t/\tau}\right)$

- **RL Circuits** $\tau = \frac{L}{R}$ = time const.

Current Growth: $I(t) = \frac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right)$, Current Decay: $I(t) = I_0\,e^{-t/\tau}$

- **LC Circuits** $\omega_0 = \frac{1}{\sqrt{LC}}$, $Q(t) = Q_{max}\cos(\omega_0 t+\phi)$

- **LCR Circuits** $Q(t) = Q_0\,e^{-\gamma t}\cos(\omega t+\phi)$, $\gamma = \frac{R}{2L}$, $\omega = \sqrt{\omega_0^2-\gamma^2}$

- **General AC Circuits** $V_{rms} = \frac{1}{\sqrt{2}}V_{max}$, $I_{rms} = \frac{1}{\sqrt{2}}I_{max}$

$\omega = 2\pi f$, $I(t) = I_{max}\cos\omega t$, $V(t) = V_{max}\cos(\omega t+\phi)$

$Z = \frac{V_{max}}{I_{max}} = \frac{V_{rms}}{I_{rms}}$ (Impedance), $\mathcal{P}_{ave} = V_{rms}\,I_{rms}\cos\phi$ (Ave. Power)

	Z	$\boldsymbol{\phi}$
Just R	R	0
Just C	$X_C = \frac{1}{\omega C}$	$-90° = -\frac{\pi}{2}$
Just L	$X_L = \omega L$	$90° = \frac{\pi}{2}$

- **Series RCL** $Z = \sqrt{R^2+(X_L-X_C)^2}$, $\tan\phi = \frac{X_L-X_C}{R}$, $\mathcal{P}_{ave} = I_{rms}^2 R$

Resonance: $Z = Z_{min} = R \iff X_L = X_C \iff \omega = \frac{1}{\sqrt{LC}}$

- **Transformer** $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$

- **Electromagnetic Radiation in Vacuum**

$\vec{E} = \hat{y}\,E_{max}\cos(k\,x-\omega\,t)$, $\vec{B} = \hat{z}\,B_{max}\cos(k\,x-\omega\,t)$

$\omega = 2\pi f = \frac{2\pi}{T}$, $k = \frac{2\pi}{\lambda}$, $f\lambda = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $c = \frac{E_{max}}{B_{max}} = \frac{E}{B}$

Intensity $= I = \frac{\text{Power}}{\text{Area}} = \frac{U}{A\,\Delta t}$, $I = \frac{E_{max}^2}{2\mu_0 c} = c\,u_{ave} = S_{ave}$, $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

- **Radiation Pressure and Momentum**

Momentum carried by radiation: $p = \frac{U}{c}$, Pressure $= \frac{\text{Force}}{\text{Area}}$

	Momentum to Surface	Pressure on Surface
Perfect Absorber	$p = \frac{U}{c}$	$P = \frac{I}{c}$
Perfect Reflector	$p = 2\,\frac{U}{c}$	$P = 2\,\frac{I}{c}$
κ = fraction refl.	$p = (1+\kappa)\,\frac{U}{c}$	$P = (1+\kappa)\,\frac{I}{c}$

- **In a Medium** $f\lambda = v = c/n$

At interface: $n_1\lambda_1 = n_2\lambda_2$
Snell's Law: $n_1\sin\theta_1 = n_2\sin\theta_2$

Total Internal Refl.: $\theta_1 > \theta_c$ where $\sin\theta_c = \frac{n_2}{n_1}$

- **Geometric Optics** $\frac{1}{f} = \frac{1}{s}+\frac{1}{s'}$, $m = \frac{h'}{h} = -\frac{s'}{s}$

Sph. Mirrors: $f = \frac{R}{2}$, $R>0$ (concave), $R<0$ (convex), $R\to\infty$ (flat)

Thin Lenses: $f>0$ (converging), $f<0$ (diverging)

Spherical Interface: $\frac{n_1}{s}+\frac{n_2}{s'} = \frac{n_2-n_1}{R}$, $m = \frac{h'}{h} = -\frac{n_1}{n_2}\frac{s'}{s}$

Flat Interface: $\frac{s'}{s} = -\frac{n_2}{n_1}$, $m = \frac{h'}{h} = 1$

Lensmaker Formula: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

- **Interference and Diffraction**

$\tan\theta = \frac{y}{L}$, Small θ or $y \ll L \implies \sin\theta = \frac{y}{L}$

Double Slit: Intensity: $I = I_{max}\cos^2\left(\frac{\pi\,d\sin\theta}{\lambda}\right)$

Const. Int.: $d\sin\theta = m\lambda$, Dest. Int.: $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$

Diffraction Grating: Const. Int: $d\sin\theta = m\lambda$, Dest. Int. elsewhere

Single Slit: Destr. Int.: $a\sin\theta = m\lambda$, $m \neq 0$

Thin Films	Constructive Interference	Destructive Interference
$n < n'$	$2t = m\,\frac{\lambda}{n}$	$2t = \left(m+\frac{1}{2}\right)\frac{\lambda}{n}$
$n > n'$	$2t = \left(m+\frac{1}{2}\right)\frac{\lambda}{n}$	$2t = m\,\frac{\lambda}{n}$

Phase Shift on Reflection: $n < n' \implies 180°$ shift, $n > n' \implies$ no shift

- **Polarization** $I = \frac{1}{2}I_0$, $I = I_0\cos^2\theta$, $\tan\theta_P = \frac{n_2}{n_1}$ (Pol. ⊥)