University Physics II Laboratory
Phys 2126 – Blinn College
Lab Manual

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Electric Fields

Equipment and Setup: Mathematica file – ElectricFields.nb

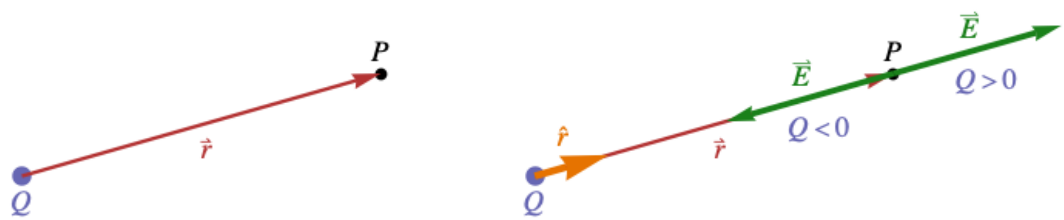
Theory

Electric Fields and Discrete Distributions

The electric field $\vec{E}$ at point P due to a point charge Q is

$$\vec{E} = k_e \frac{Q}{r^2}\hat{r}$$

As shown in the figure below, $\vec{r}$ is the vector from the charge Q to the point P, $\hat{r} = \vec{r}/r$ is the unit vector in the direction of $\vec{r}$ and r is the magnitude of the vector $\vec{r}$, the distance from the charge. The field points away from the charge when Q is positive and toward the charge when Q is negative.



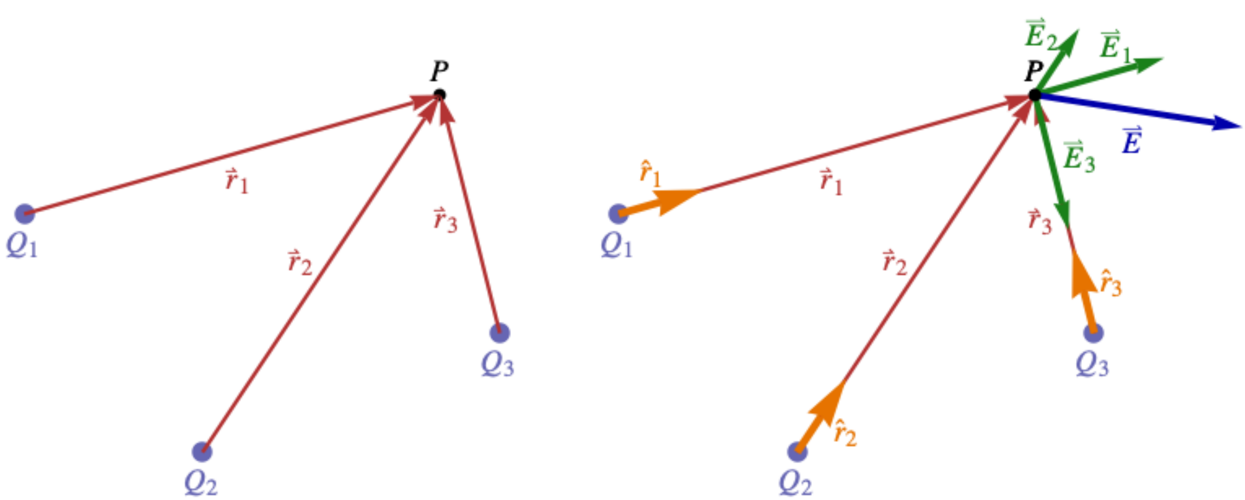
The magnitude of the electric field is $E = k_e|Q|/r^2$; the absolute value of the charge is needed to guarantee that the magnitude is positive. E gets smaller with increasing distance from the charge. The constant k_e is related to the more fundamental constant ε_0. It has the value

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \, \frac{\text{N·m}^2}{\text{C}^2}$$

A discrete distribution of charge is a collection of point charges $Q_1, Q_2, \cdots$. Label the i^{th} charge as Q_i and the field due to the i^{th} charge as $\vec{E}_i$. To calculate the field of a discrete distribution at some point P, sum the electric field vectors due to each charge in the distribution:

$$\vec{E} = \sum_i \vec{E}_i = k_e \sum_i \frac{Q_i}{r_i^2}\hat{r}_i$$

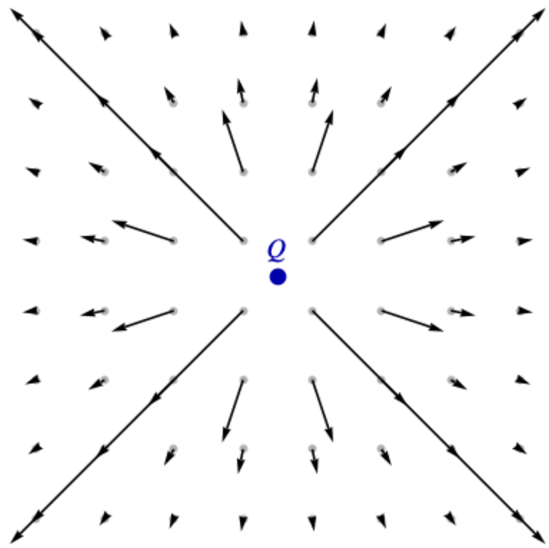
Here $\vec{r}_i$ is the vector from Q_i to P; it has magnitude r_i and the unit vector in its direction is $\hat{r}_i$, where $\hat{r}_i = \vec{r}_i/r_i$, as shown in the figure below.



A three-charge discrete distribution. From the directions of the fields, it follows that Q_1 and Q_2 are positive and Q_3 is negative.

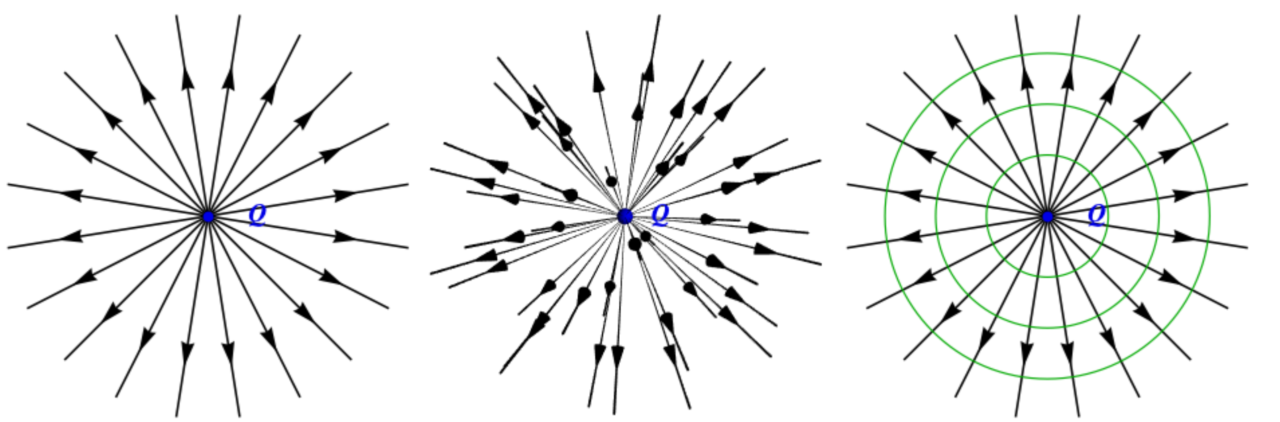
Visualization of Electric Fields

The electric field is an example of a vector field. A vector field is a vector-valued function of position, meaning that at each point in space, the vector has a value that can vary with position. A common way to represent vector fields is to show the vector at each point on a grid. Here is such a diagram for the field due to a positive point charge.



This is a mess even for the simple case of a single point charge. For more complex distributions of charges, such a diagram does not help us visualize the field at all.

What we do instead to visualize electric fields is draw continuous curves showing the direction of the field. For a point charge we get:



Left: Field lines for a point charge in 2D Center: 3D version of the same Right: Concentric spheres around the charge

We can recover the information about the magnitude of the field. If you add different spheres centered at the charge to this, as shown at the right of the diagram above, then the number of field lines passing though each sphere is the same. Since the area of a sphere is proportional to r^2, it follows that the number of lines per area is proportional to $1/r^2$. The electric field is also proportional to $1/r^2$ and this gives our graphical interpretation of the field strength E from a field diagram.

$$E \propto \frac{\text{number of field lines}}{\text{area}}$$

This also applies to more general field diagrams. When the lines are closer the field is stronger and when further apart it is weaker. The absolute number of lines drawn in a diagram is unimportant; it is the relative number of lines at a position that matters.

In this experiment, we will explore field diagrams for discrete distributions with consisting of two charges. The diagrams of field lines are drawn using Mathematica. The algorithm for drawing these lines is simple. Near a point charge, the field due to that charge will dominate that of any other charges in the distribution; this is because the *r* in the denominator will be very small, making that field huge. Begin with field lines from one charge directed radially outward. For each field line, move a small distance in the direction of the field and then calculate the new field at that position and move another small distance in that direction. Continue this until the field line hits another charge or moves far enough out to be beyond the diagram. The electric field at any point is tangent to the field lines and the density of lines, the number per area, will be related to the field strength.

Motion of Charged Particles in Electric Fields

The force on a charge q in an electric field $\vec{E}$ is

$$\vec{F}_{\text{elec}} = q\vec{E}$$

If this electric force is the only force acting then it is the net force $\vec{F}_{\text{net}} = \vec{F}_{\text{elec}}$ and we can then use Newton's second law to find the acceleration of the particle:

$$\vec{F}_{\text{elec}} = q\vec{E} = m\vec{a}$$

Section B of this experiment is a simulation of shooting charged particles into a region with a uniform electric field. The uniform field will give rise to a constant force and thus a constant acceleration. The Physics I diccussion of kinematics for the case of constant acceleration will apply here. The fields will be left-right, so the resulting trajectories will look like projectiles, rotated by $\pm 90°$.

The particles will be shot into a region of uniform electric field with an initial velocity directed toward the top of the screen. The particles will consist of the electron e^-, the proton p, the neutron n, the alpha particle α and the positron e^+. The proton and neutron have approximately the same mass and are much more massive than the electron. The alpha particle is the nucleus of helium-4, which has 2 protons and 2 neutrons. The positron is the antiparticle of the electron; as an antiparticle, it must have the same mass and opposite charge. The masses and charges of these particles are listed in the interactive panel and are also summarized here:

	Mass	Charge
electron: e^-	9.109×10^{-31}kg	$-e$
proton: p	1.673×10^{-27}kg	$+e$
neutron: n	1.675×10^{-27}kg	0
alpha particle: α	6.645×10^{-27}kg	$+2e$
positron: e^+	9.109×10^{-31}kg	$+e$

The charges are written in terms of the elementary charge *e*:

$$e = 1.602 \times 10^{-19}$$

Electric Fields – Worksheet Name________________ Group______

Section A: Electric Fields Due to Two Charges

Computer Setup for Section A

- The first interactive panel shows electric fields due to two point charges, Q_1 at $(-1\text{ m}, 0)$ and Q_2 at $(1\text{ m}, 0)$. The controls for this panel are at the top on the left.
- The top line has two checkboxes: one to Show Axes and the other to Show Field Lines. The top line also has a slider labeled "Scale Factor"; this rescales the electric field vector arrows relative to the drawing. Click the checkboxes and move the slider to see what happens. To undo any changes, click the Reset button on the upper right of the panel.
- The next line has buttons with three different values of the charges. Click these to see what happens.
- The third line selects the point where the fields are evaluated. The five buttons give preset values for the *x*- and *y*-coordinates of this point. You can also drag the "locator" (crosshairs) to move the evaluation point to any position. Try this.
- At the right of the control panel, the *x*- and *y*-components of the fields are shown. The vectors $\vec{E}_1$ and $\vec{E}_2$ are the electric fields due to Q_1 and Q_2, respectively; these are displayed with the green arrows in the picture. $\vec{E}$ is the total field, the vector sum of $\vec{E}_1$ and $\vec{E}_2$.

Data Recording

1. Select the charge configuration $Q_1 = -2\mu\text{C}$ and $Q_2 = +2\mu\text{C}$ (called a *dipole*) and check Show Axes. Drag the locator to move the evaluation position along the *y*-axis. In the space below, **describe the direction of the field along the *y*-axis.**

2. Select the second charge configuration ($Q_1 = -2\mu\text{C}$ and $Q_2 = +3\mu\text{C}$), check Show Axes and select the position $(0,1\text{ m})$. In the space below, **record** $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}$. These electric field vectors will be calculated in a later question.

3. Select the third charge configuration ($Q_1 = -2\mu\text{C}$ and $Q_2 = -3\mu\text{C}$). With Show Axes checked drag the locator to a position along the *x*-axis between the two charges. Find the position where the electric field becomes zero. **Record this position** in the space below. (It suffices for the components to be less than 0.5×10^3 N/C, which is sufficiently small compared to $\vec{E}_1$ and $\vec{E}_2$.) This position will be calculated in a later question.

Section B: Trajectory of a Charged Particle in a Uniform Electric Field

Computer Setup for Section B

- Now scroll down to the second interactive panel. This shows the paths of various charged particles that are shot into a region of uniform electric field. The field points left-right; a positive value of E_x corresponds to a field to the right. The particle is shot in the +*y*-direction into this field with initial speed v_0.
- At the top right are two buttons, a reset button and a U-shaped update button. The left of the control panel allows for the selection of the particle. The choices are: electron, proton, neutron, alpha particle and positron.
- In the middle of the control panel you can choose the value of the initial speed v_0 and the electric field E_x. Anytime you change these values, you should then click the update button at the upper right.
- There is also a checkbox to Animate Motion. Checking this box shows controls for animation. This may slow things down too much; if so, uncheck it.
- The Exit Data is listed below the control panel.

Data Recording

Use $v_0 = 600{,}000$ m/s and $E_x = 15$ N/C for Steps 1 through 3 below.

1. Select the electron e^- and record the exit data.

   $x_f =$ ______________ cm, $y_f =$ ______________ cm, $t_f =$ ______________ ns

2. Select the positron e^+ and record the exit data.

   $x_f =$ ______________ cm, $y_f =$ ______________ cm, $t_f =$ ______________ ns

3. Select the proton *p* and record the exit data.

   $x_f =$ ______________ cm, $y_f =$ ______________ cm, $t_f =$ ______________ ns

4. Experiment with different values of E_x, keeping $v_0 = 600{,}000$ m/s, to find the value needed for the proton to land at the same position as the electron (in Step 1). Record this value of E_x below.

   $E_x =$ ________________ N/C

5. Experiment with different values of v_0, using $E_x = 15$ N/C, to find the value of v_0 needed for the proton to land at the same position as the positron (in Step 2). Record below.

   $v_0 =$ ________________ m/s

Questions

A-1. Explain why the field of the dipole is perpendicular to the *y*-axis, as observed in Section A, Step 1.

A-2. Derive the result in Section A, Step 2.

A-3. Derive the result in Section A, Step 3. **Hint:** For the field $\vec{E}$ to be zero the vectors $\vec{E}_1$ and $\vec{E}_2$ must be equal in magnitude and opposite in direction. Since both charges are negative, the fields $\vec{E}_1$ and $\vec{E}_2$ point toward the corresponding charges. The only points where the fields are in opposite directions are points between the two charges on the *x*-axis. This gives:

$$k_e \frac{|Q_1|}{r_1^2} = k_e \frac{|Q_2|}{r_2^2}$$

Use this equation to find the *x*-coordinate of the position where the net field is zero.

B-1. Compare the paths in Section B, Steps 1 and 2. Explain the differences and similarities.

B-2. Calculate the components of the acceleration vector $\vec{a}$ for both the electron and positron in Section B, Steps 1 and 2. **Record the acceleration of each particle in component form below.**

Electron: $\vec{a}$ = ______________________ Positron: $\vec{a}$ = ______________________

B-3. Calculate the speed of the electron as it leaves the screen in Section B, Step 1.

B-4. Why is the proton's deflection in Section B, Step 3 so small?

B-5. Explain the results of Section B, Steps 4 and 5.

Electric Potential and Conductors

Equipment and Setup: Mathematica file – ElectricPotential.nb

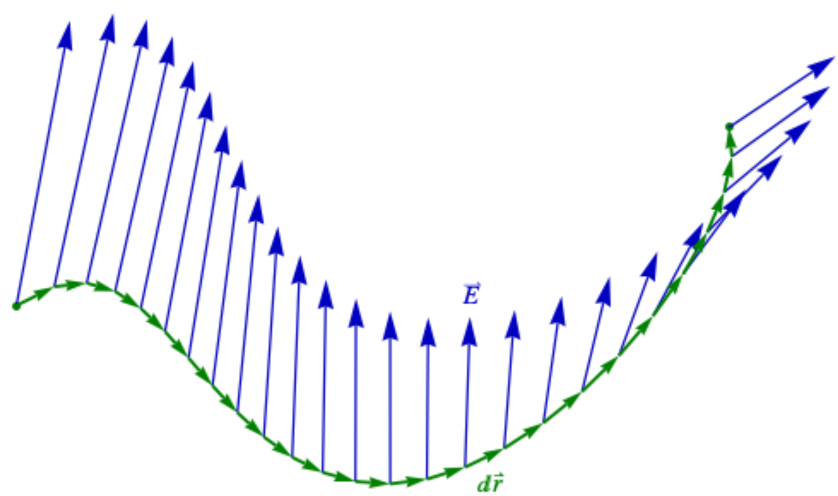
Theory

Electric Potential and Discrete Distributions

The electric potential V is a scalar quantity. It is related to the electric potential energy U by $\Delta U = q\Delta V$, where ΔU is the change in potential energy when a charge q is moved across a potential difference ΔV. Just as the choice of zero for potential energy is arbitrary, the choice of zero for electric potential is arbitrary as well. We can also relate the electric potential V to the electric field $\vec{E}$. When moving along a path, the potential difference between the path's endpoints is given by integrating the electric field along the path:

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

The diagram below shows a varying electric field, where $d\vec{r}$ is an infinitesimal displacement along the path.



A varying electric field is integrated along a path to get ΔV, the change in potential.

The discussion from the previous experiment considered the electric fields for a point charge and for a discrete distribution of charge. Now we will do the same for electric potential. Without proving the details, when the field for a point charge is integrated, as described above, we get the potential of a point charge:

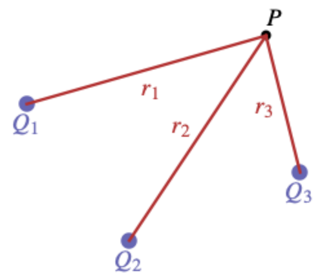
$$V = k_e \frac{Q}{r}$$

With this expression we have made the choice of zero for the potential; we take V to be zero at infinity, meaning that $\lim_{r\to\infty} V = 0$. Here r is the distance from the charge Q to the point P. Since the distance r must be positive, the sign of the potential V is the sign of the charge Q.

For a discrete distribution, as we did for electric fields, we sum over the electric potentials due to each charge in the distribution. It is much easier to find V for a distribution than the corresponding calculation for electric fields because we are simply summing scalars and not vectors:

$$V = k_e \sum_i \frac{Q_i}{r_i}$$

The diagram below shows a discrete distribution with three point charges.

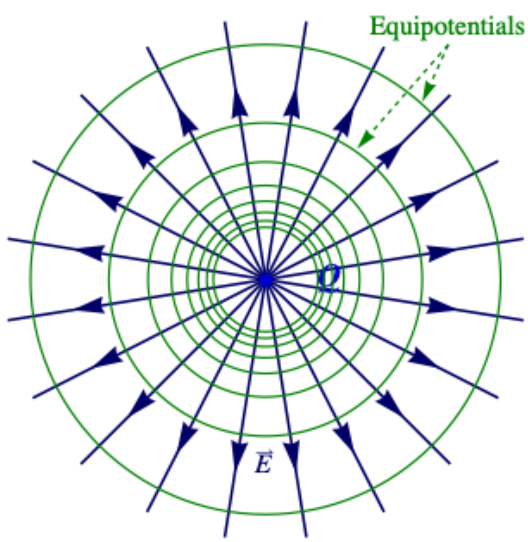


The electric potential *V* at point *P* due to a discrete distribution. r_i is the distance from Q_i to *P*,

Visualization and Equipotentials

The first part of the previous experiment emphasized the visualization of electric fields using electric field lines. These were continuous curves showing the direction of the field at all positions. We will now add to these field diagrams a way of visualizing the electric potential *V*. The potential is a scalar field, meaning that it is a scalar function of position. There is a standard way to visualize scalar fields. Suppose you are considering temperature (a scalar field) on a weather map. On the weather map we can draw curves of constant temperature called *isotherms*. Similarly, on a weather map we can represent pressure, another scalar field, by drawing curves of constant pressure, called *isobars*. An example, from geology, is a topographical map. We visualize elevation as a function of horizontal position by drawing contours of constant elevation.

The analogous things for electric potential are equipotentials. These are, in a two-dimensional diagram, contours of constant potential and in three dimensions, surfaces of constant potential. Let us first consider the simple case of a point charge. For that, the potential is $V = k_e Q/r$, so a surface of constant *V* is a surface of constant *r*, a sphere. The equipotential surfaces for a point charge are concentric spheres with the charge at the center.



The equipotentials for a point charge are concentric spheres.

Note that in the diagram above for a point charge, the equipotentials are perpendicular to the field lines. We will see that this is generally the case. To show this, consider the infinitesimal form of $\Delta V = -\int \vec{E} \cdot d\vec{r}$:

$$dV = -\vec{E} \cdot d\vec{r}$$

Here dV is the infinitesimal change in the potential *V* when moving along the infinitesimal displacement $d\vec{r}$. Choose $d\vec{r}$ to point in some direction along an equipotential surface. Since *V* is uniform on the surface, $dV = 0$ and that implies that $\vec{E} \cdot d\vec{r} = 0$. When the dot product of two non-zero vectors is zero, the vectors are perpendicular. It follows then that $\vec{E}$ is perpendicular to *any* direction along an equipotential, and therefore must be perpendicular to the equipotential surface itself.

Also, using the expression $dV = -\vec{E} \cdot d\vec{r}$, now choose $d\vec{r}$ to be in the direction of the field. Since the scalar product (dot product) of two vectors in the same direction is positive, then it follows that $dV < 0$. This tells us that in electrostatics, the electric field always points toward lower potential.

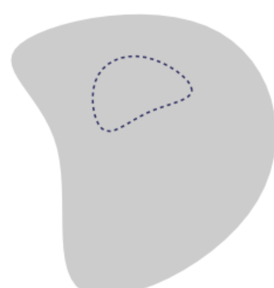
Summarizing facts about equipotentials in electrostatics:

- An equipotential is a surface (or a contour in a two-dimensional figure) of equal potential V.
- Equipotentials are perpendicular to electric field lines.
- Electric field lines point toward lower potential.

Conductors in Electrostatics

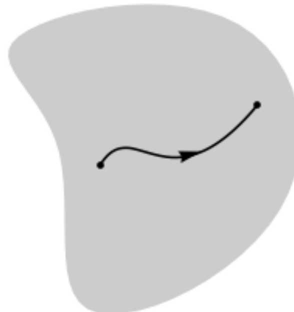
Inside a conductor there are charge carriers, which are freely moving charges. The most typical conductors are metals where the charge carriers are electrons. Most electrons in a metal are bound to their atom but one or a few of the electrons are not tied to their atom but are shared in a *sea of electrons*, which can move freely. In a semiconductor the conduction mechanism is different but there are still charges, which could be either positive or negative, that are free to move. If there is an electric field inside a conductor then the freely moving charges will move and that will give electric currents. The central assumption of electrostatics, however, is that nothing can move; there can be no currents in electrostatics. It follows then that in electrostatics, the electric field must be zero everywhere inside a conductor.

Gauss's law relates the electric flux through a closed surface to the total electric charge enclosed by that surface: $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\varepsilon_0$. If we apply this to a Gaussian surface entirely inside a conductor, as shown in the figure below, then the left-hand side of Gauss's law is zero; this is because the electric field vanishes, and that implies that $Q_{\text{enclosed}} = 0$. This gives an important result: there is no excess charge inside a conductor. Any excess charge must be at the surface of a conductor.



The dashed curve represents a Gaussian surface entirely inside a conductor.

Now consider the integral $\Delta V = -\int \vec{E} \cdot d\vec{r}$ along a contour entirely inside a conductor, as shown in the figure below. Because the electric field is zero, $\Delta V = 0$. This means that the electric potential is uniform throughout a conductor in electrostatics.



Integrating the electric field along a contour entirely inside a conductor, implies that V is constant throughout a conductor.

Since the potential is uniform throughout a conductor, the surface of a conductor must be an equipotential. From this, we can conclude that the electric field at the surface of a conductor is perpendicular to that surface.

Since all the excess charge is at the surface of a conductor, we can describe the distribution of charge with a surface charge density σ, where σ is the charge per area at some point on the surface. A Gauss's-law argument, which we will not repeat here, shows that the outward-directed component of the perpendicular electric field at some position on the surface is proportional to the surface charge density at that position.

$$E_\perp = \sigma/\varepsilon_0$$

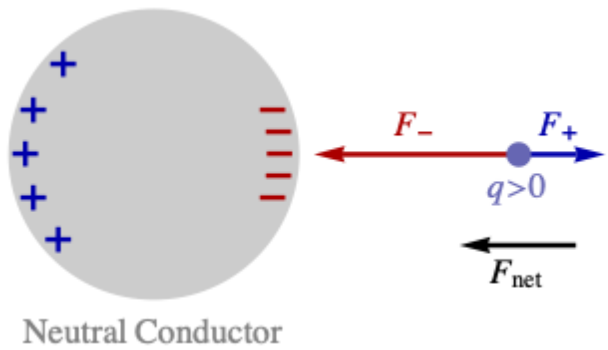
Note that σ can be positive or negative. A positive σ corresponds to an outward-pointing field and a negative σ implies and inward-pointing field.

Summarizing conductors in electrostatics:

- The electric field is zero everywhere inside a conductor.
- There is no excess charge inside a conductor. All excess charge is at the surface of a conductor.
- The electric potential V is constant throughout a conductor.
- The electric field is perpendicular to the surface of a conductor.
- The outward (perpendicular) electric field at the surface of a conductor is proportional to the surface charge density at that point $E = \sigma/\varepsilon_0$.

A Point Charge Near a Conductor

When a positive point charge is brought near a neutral conductor, the electric field of the point charge will redistribute the charge in the conductor. There will be an accumulation of negative charge near the positive point charge and, since the conductor is neutral, there must be an equal amount of positive charge accumulating away from the point charge. The negative charge will give an attractive force on the point charge and the positive charge will give a repulsive force. Since the positive charge is farther away than the negative charge, the net force is attractive, as shown in the figure below.



A point charge, whether positive or negative, is always attracted to a neutral conductor. For a negative point charge the preceding argument applies if all the signs are reversed.

If the conductor is given a net positive charge, then it will repel a positive point charge when a long distance from it. However, if the point charge is close enough to the conductor, then the negative built-up charge on the conductor near the point charge will be sufficient to create a net attractive force. This behavior will be studied in the second part of this experiment.

Electric Potential – Worksheet Name________________ Group______

Section A: Electric Potential Due to Two Charges

Computer Setup for Section A

- The first interactive panel shows different representations of the electric potential due to two point charges: Q_1 at $(-1\text{m}, 0)$ and Q_2 at $(1\text{m}, 0)$. The buttons at the top left allow for choosing between three different sets of charges. The buttons at the top right gives a choice between two different ways to display the electric potential: the Interactive 2D Plot button shows equipotentials and electric field lines and (x,y,V) Plot shows a 3D display of V as a function of x and y, with the equipotentials drawn in. Click through to see both style displays for each set of charges. There is a small reset button at the upper right.
- In the Interactive 2D Plot display there is a second set of controls that appear. There are three checkboxes: one to Show Axes, another to Show Field Lines and a third to Show $\vec{E}$. The top line also has a slider labeled "$\vec{E}$ Scale Factor"; this rescales the electric field vector arrows relative to the drawing. Click the checkboxes and move the slider to see what happens. To undo any changes here, click the other Reset button on the upper right of this inside panel. Note that holding the mouse over an equipotential gives its value in Volts.
- The next line selects the point where the potentials are evaluated. The five buttons give preset values for the x- and y-coordinates of this point. You can also drag the "locator" (crosshairs) to move the evaluation point to any position. Try this.
- At the right of the control panel, the electric potentials are shown. The values V_1 and V_2 are the electric potentials due to Q_1 and Q_2, respectively, and V is the sum of the two.

Data Recording for Section A

1. Select the charge configuration $Q_1 = -2\ \mu\text{C}$ and $Q_2 = +2\ \mu\text{C}$ (called a *dipole*) and check Show Axes. Drag the locator to move the evaluation position along the y-axis. Notice that the y-axis is the zero equipotential. View this in the (x,y,V) Plot display as well.
2. Select the second charge configuration ($Q_1 = -2\ \mu\text{C}$ and $Q_2 = +3\ \mu\text{C}$), check Show Axes and select the position $(-1\text{ m}, -2\text{ m})$. In the space below, **record** V_1 and V_2 and V. These electric potentials will be calculated in a later question.

3. Continuing with the charge configuration ($Q_1 = -2\ \mu\text{C}$ and $Q_2 = +3\ \mu\text{C}$), check Show Axes. Holding the mouse over the equipotentials identify the zero equipotential. Estimate the positions along the x-axis where the potential is zero. Dragging the locator along the equipotential may help in doing this. These positions will be calculated in a later question.

4. Select the third charge configuration ($Q_1 = -2\ \mu\text{C}$ and $Q_2 = -3\ \mu\text{C}$). Are there any positions where the potential is zero?

Section B: Point Charge Near a Conducting Sphere

Computer Setup for Section B

- Now scroll down to the second interactive panel. This shows a movable positive point charge, the blue dot, near a conducting sphere, the gray disk. The surface charge densities on the conducting sphere are shown in red for negative and blue for positive.
- Moving from left to right along the top line of controls we find a slider that changes x_0, the position of the point charge, the value of x_0 and a pair of buttons that select between a neutral conductor, Zero Net Charge and a positively charged conductor, Positive Net Charge.
- The second line of controls has three checkboxes and a slider. The Show Image Charges checkbox allows you to see the (hidden) trick used to draw these field and potential configurations; it is known as the Method of Images. We will not go deeper into this trick.
- There is also a checkbox to Show F which shows the force vector on the point charge. The slider F Scale allows you to resize the vector when it becomes too large or small. The vector value of the force is shown to the right.

Data Recording for Section B

1. For the case of Zero Net Charge on the conductor vary the position and describe how the force changes. Describe how the surface charge densities change as well.

2. For the case of Positive Net Charge on the conductor vary the position and describe how the force changes and also describe how the surface charge densities change.

Questions

A-1. Explain why for the dipole the *y*-axis is the zero equipotential, as observed in Section A, Step 1.

A-2. Derive the result in Section A, Step 2.

A-3. Calculate the positions along the *x*-axis where the potential is zero. Compare this to what you found in Section A, Step 3.

A-4. Why is there nowhere, other than infinity, where the potential is zero in Section A, Step 4?

B-1. Applying Coulomb's law $F = k_e|q_1||q_2|/r^2$ to the case of the neutral conductor, we have one positive charge, the point charge, and one zero charge, the conductor; this should give zero force, which is incorrect here. What is wrong with this naive application of Coulomb's law here? Explain.

B-2. Explain why the force on the point charge should always be toward the neutral conductor.

B-3. For the case of the conductor with the Positive Net Charge, why should the force on the point charge be attractive when close and repulsive when far?

Name ________________ Group ______

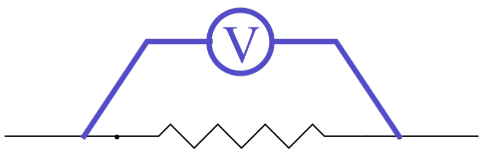
Ohm's Law

Equipment and Setup: Circuit board, Multimeters (2)

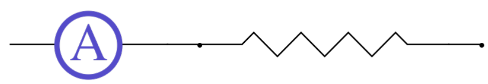
Using the Multimeter

We will use a multimeter to measure voltage, current and resistance. The black lead to the multimeter should always be kept in the black socket. When using it as a voltmeter or Ohmmeter the red lead should be in the V-Ω socket. When using it as an ammeter the red lead should be set to one of the two ammeter sockets.

When making reading with a multimeter always use the lowest scale that reads. The number on the scale is the maximum reading of that scale. If there is a metric multiplier in a scale then you multiply the result by that multiplier. For example, if using an ammeter in the 200 mA scale the meter reads 153, then the result is 153 mA.



Use of Voltmeter

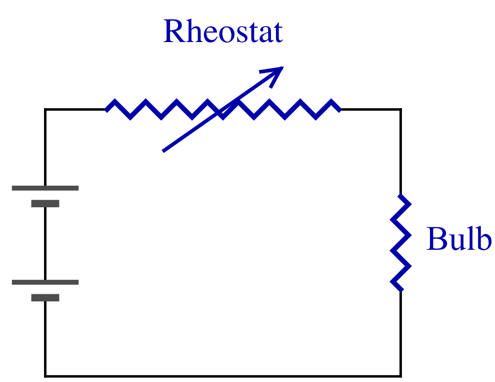


Use of Ammeter

(A) Non-ohmic Behavior

Using a series circuit with both batteries, the rheostat and a light bulb demonstrate the non-ohmic behavior of a light bulb. Make a reading of voltage and current for each of five different setting of the rheostat. Use the 10 A setting on the ammeter to measure the current.

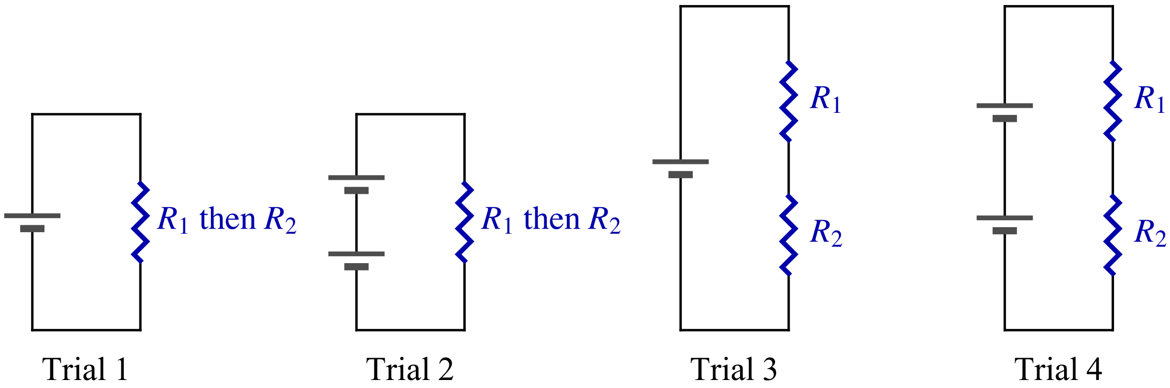
Voltage across bulb V (V)	Current through bulb I (A)	V/I (Ω)



Using software plot a graph of Voltage vs. Current.

Question: Does your graph show Ohmic behavior? Explain. Discuss why a light bulb should *not* be Ohmic.

(B) Variation of Current with Voltage for Fixed Resistance



Select two different resistors between 100 Ω and 1000 Ω. For each resistor make four measurements of voltage and current. Keep $R_1 < R_2$ and $R_2 < 3\ R_1$. The diagrams above show how to get four different values of V and I for each of our two resistors. When measuring V make sure you are measuring the voltage across *just* that resistor. Note that both currents in Trial 3 (and also for Trial 4) should be the same.

	R_1 = ______________			R_2 = ______________		
Trial	Voltage across R_1 V_1 (V)	Current through R_1 I_1 (A)	Calculate V/I (Ω)	Voltage across R_2 V_2 (V)	Current through R_2 I_2 (A)	Calculate V/I (Ω)
1						
2						
3						
4						

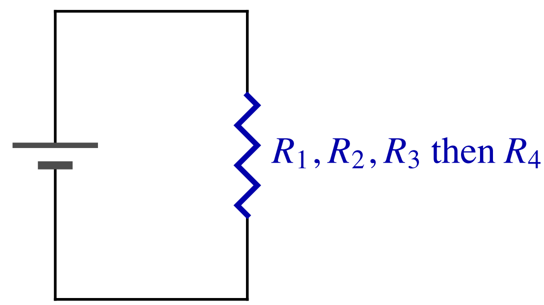
Using software plot a graph of Voltage vs. Current for each resistor. There can be two separate graphs or both plots could be on the same graph. For each resistor plot the best-fit line, the equation of the best-fit line and find its slope. Compare the slopes with the resistances. Give the percent errors.

(C) Variation of Current with Resistance for Fixed Voltage

Select an additional two different resistors between 100 Ω and 1000 Ω. Use a single battery and just one resistor at a time. The voltage of the battery is the Fixed Voltage. Measure the current through each resistor.

Fixed Voltage = ____________________

Resistance R (Ω)	Current I (A)	Resistance^{-1} $1/R$ (Ω^{-1})



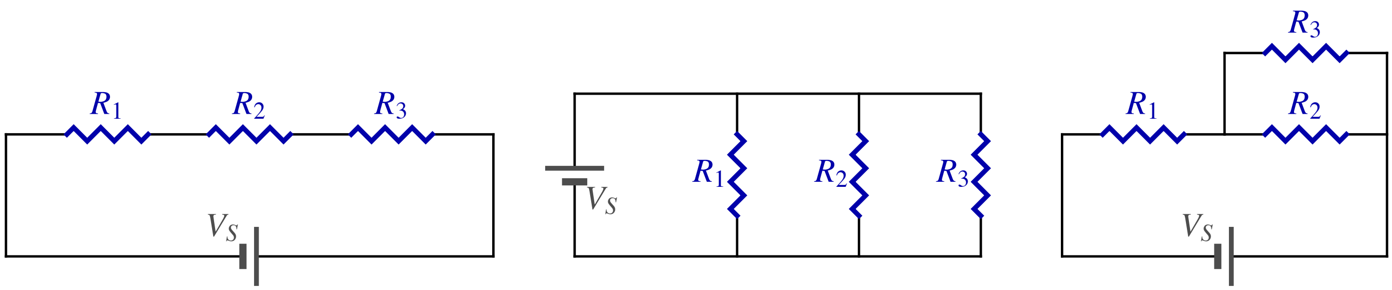
Using software plot a graph of. Current vs. $1/R$. Plot the best-fit line, the equation of the best-fit line and find its slope. Compare the slope with the Fixed Voltage and give the percent error.

Name ________________ Group ______

Series and Parallel Circuits

Equipment and Setup: Circuit board, Multimeters (2)

The three circuits we will consider are series, parallel and series-parallel.



Series Parallel Series−Parallel

Recall that when measuring voltage across something, the voltmeter should be connected in parallel with it. When measuring current through something, the meter should be placed in series with it. (Suggestion: When measuring the current through a resistor it is sometimes difficult to isolate that resistor for the measurement. To do this, remove one end of the resistor and put the meter between the loose end and its connection.)

Procedure

Select three resistors, in increasing order, between 100 Ω and 1000 Ω. From the stripes on the resistors, read the listed values of their resistances. Compare with the measured values obtained from the multimeter, set as an Ohmmeter.

	Listed Values (from stripes)	Measured Values (from meter)	% Error
R_1			
R_2			
R_3			

Use the same battery for each part of this experiment. Measure the voltage across this battery. Call this the Source Voltage.

Source Voltage: V_S = _______________

Use the Source Voltage in all your theoretical calculations of current and voltage. Also use the listed values of the resistances for all calculations.

(A) Series Circuit

Connect the three resistors in series, measure their equivalent resistance and compare with the theoretical value. Use the listed values in your calculations.

	Measured Value (from meter)	Theoretical Value (from calculation)	% Error
Equivalent Resistance: R_{eq}			

Calculation (show work)

Connect this series arrangement across a single battery. Measure the current and the voltages across each resistor.

	Measured Value (from meter)	Theoretical Value (from calculation)	% Error
Total Current: I_{tot}			
Voltage Across R_1: V_1			
Voltage Across R_2: V_2			
Voltage Across R_3: V_3			

Calculations (show work)

Find the sum of the measured voltages: $V_1 + V_2 + V_3 =$ ________________

Compare this with the Source Voltage. % difference = ________________

(B) Parallel Circuit

Connect the three resistors in parallel, measure their equivalent resistance and compare with the theoretical value. Use the listed values in your calculations.

	Measured Value (from meter)	Theoretical Value (from calculation)	% Error
Equivalent Resistance: R_{eq}			

Calculation (show work)

Connect this parallel arrangement across a single battery. Measure the total current and the currents through each resistor.

	Measured Value (from calculation)	Theoretical Value (from calculation)	% Error
Total Current: I_{tot}			
Current Through R_1: I_1			
Current Through R_2: I_2			
Current Through R_3: I_3			

Calculations (show work)

Find the sum of the measured currents: $I_1 + I_2 + I_3 =$ ________________

Compare this with the measured Total Current, I_{tot}. % difference = ________________

(C) Series-Parallel Circuit

Connect R_2 and R_3 in parallel and them connect this in series with R_1. Measure their equivalent resistance and compare with the theoretical value. Use the listed values in your calculations.

	Measured Value (from meter)	Theoretical Value (from calculation)	% Error
Equivalent Resistance: R_{eq}			

Calculation (show work)

Connect this arrangement across a single battery. Measure the currents through each resistor and the voltages across each resistor.

	Measured Value (from meter)	Theoretical Value (from calculation)	% Error
Current Through R_1: $I_1 = I_{tot}$			
Current Through R_2: I_2			
Current Through R_3: I_3			
Voltage Across R_1: V_1			
Voltage Across R_2 & R_3: $V_2 = V_3$			

Calculations (show work)

Find the sum of the measured voltages: $V_1 + V_2 =$ _______________

Compare this with the Source Voltage. % difference = _______________

Find the sum of the measured currents: $I_2 + I_3 =$ _______________

Compare this with the measured current through R_1, $I_1 = I_{tot}$. % difference = _______________

Question 1 Suppose the resistors in the various circuits were light bulbs. When a bulb burns out it becomes a break in the circuit: this is equivalent to an infinite resistance. In each of the three circuits used, what happens to one bulb when another burns out. Specify whether the bulb goes dark, stays the same brightness, burns brighter or burns more dimly. Assume the battery is an ideal source with no internal resistance.

(a) If a bulb in the series circuit of part (A) burns out, what happens to the other bulbs.

(b) If a bulb in the parallel circuit of part (B) burns out, what happens to the other bulbs.

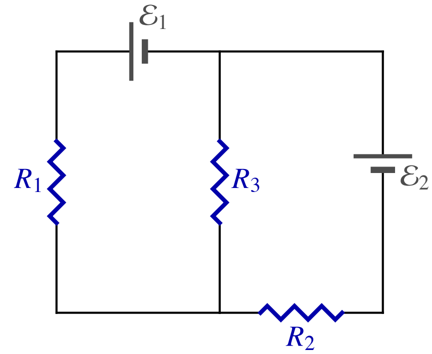
(c) If in the series-parallel circuit of part (C) if the bulb for R_2 burns out, what happens to R_1 and R_3?

Question 2 Show that there are 17 different equivalent resistances that can be formed using three resistors. (Use one of three, two of three or all three.)

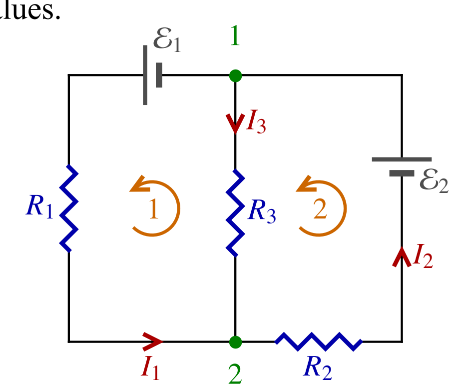
Kirchhoff's Rules

Equipment: Circuit board, 2 Multimeters, Mathematica file – Kirchhoff.nb

Theory: We will consider circuits with several resistors and multiple dc voltage sources. We will refer the voltage sources as EMFs (electromotive force) and label them with the symbol $\mathcal{E}$. Kirchhoff's rules give a procedure to find the current in each branch of the circuit in terms of the resistances and EMFs. As a simple example of such a circuit, consider the diagram below.



To find an independent set of linear equations for all the currents, we will use Kirchhoff's two rules, the junction rule and the loop rule. Before this, we must label all the currents. For each branch of the circuit label a current; in the above example there are three branches and thus there are three currents. It is not important to guess the correct direction of each current; make a choice and stay consistent with that convention. If the direction choice happens to be opposite the eventual current direction, then the value you solve for will be negative. Try to guess the direction so that the currents flow from the positive terminal (the long one) to the negative of each EMF; this will increase the chance that the eventual current is positive, but this will ultimately depend on the resistance and voltage values.



- **The Junction Rule**

A junction is some point in a circuit where three or more wires meet. Where only two wire meet, as at each corner in the above example, then that is just a bend in the wire and not a junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}} \qquad \text{(Junction Rule)}$$

To implement this for the two-loop example, first label the two junctions; here they are labeled 1 and 2. At junction 1, current I_2 enters and the currents I_1 and I_3 both leave. This gives.

$$I_2 = I_1 + I_3 \qquad \text{(Junction 1 Equation)}$$

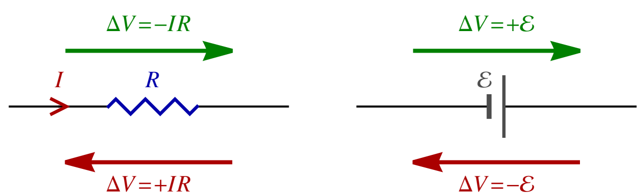
The same procedure for junction 2 gives: $I_1 + I_3 = I_2$. This is the same as the equation for junction 1 and can be omitted. For more complicated circuits, with more junctions, then there is always one too many junction rule equations. Since each current begins at one junction and ends at another, then if there are n currents then summing all the junction rule equations must give: $I_1 + I_2 + \cdots + I_n = I_1 + I_2 + \cdots + I_n$ or just $0 = 0$. We want an independent set of linear equations, so we can omit any one of our junction rule equations, because any one equation is equivalent to the sum of all the others.

- **The Loop Rule**

A loop is where you connect branches of a circuit to form a closed loop that ends where is starts. For each closed loop, the sum of all the voltage changes (gains) is zero.

$$\sum \Delta V = 0 \qquad \text{(Loop Rule)}$$

Getting the signs correct is essential. Recall that Ohm's law gives the voltage drop across a resistor when passing through it in the direction of the current. So, when passing through a resistance R in the chosen direction of the current I we get a voltage drop: $\Delta V = -IR$ and when passing through against the current it is a voltage gain: $\Delta V = +IR$. For an EMF of magnitude $\mathcal{E}$ then passing through from the negative terminal (the shorter line) to the positive gives a voltage gain $\Delta V = +\mathcal{E}$ and when passing through from positive to negative it is a voltage drop $\Delta V = -\mathcal{E}$.



For the example there are two loops, labeled 1 and 2. For loop 1 start before the EMF; you pass through $\mathcal{E}_1$ from the negative terminal to the positive, then through R_1 in the direction of the current I_1 and finally through R_3 opposite to I_3. Following the sign conventions gives:

$$0 = \mathcal{E}_1 - I_1 R_1 + I_3 R_3 \qquad \text{(Loop 1 Equation)}$$

Similar analysis on loop 2 gives:

$$0 = \mathcal{E}_2 - I_3 R_3 - I_2 R_2 \qquad \text{(Loop 2 Equation)}$$

There is another loop that could be considered, the loop that includes both of the smaller loops and traverses the perimeter of the circuit. The corresponding loop equation will just be the sum of the other two loop equations, and this not be independent. To guarantee that all the loop equations are independent, you can always choose the smallest loops.

If we had started a loop at a different point, then we would get the same terms but in a different order; this would give an equivalent equation. Traversing a loop in a clockwise sense would give the same equation with the all the terms having the opposite signs and in a different order, but still giving an equivalent equation.

- **The Solution**

These two loop equations combined with the one junction equation gives us a complete set of independent linear equations that can be solved for the currents. For this simple two-loop case we have a set of three linear equations. Usually one would be given numerical values of the voltages and resistances and the solution would be straight-forward. With symbolic variables, as this problem is stated, the algebra is messy, so we will use Mathematica to solve this analytically. This will be done in Kirchhoff.nb, the Mathematica notebook file, where you will see that the analytical solution is:

$$I_1 = \frac{\mathcal{E}_2 R_3 + \mathcal{E}_1 R_2 + \mathcal{E}_1 R_3}{R_2 R_3 + R_1 R_2 + R_1 R_3}, \quad I_2 = \frac{\mathcal{E}_1 R_3 + \mathcal{E}_2 R_1 + \mathcal{E}_2 R_3}{R_2 R_3 + R_1 R_2 + R_1 R_3} \quad \text{and} \quad I_3 = \frac{\mathcal{E}_2 R_1 - \mathcal{E}_1 R_2}{R_2 R_3 + R_1 R_2 + R_1 R_3}$$

It will also be shown in the Mathematica notebook, that if we were given the numerical values

$$\mathcal{E}_1 = 1.5\text{ V},\ \ \mathcal{E}_2 = 1.5\text{ V},\ \ R_1 = 200\ \Omega\,,\ \ R_2 = 400\ \Omega\ \text{ and }\ R_3 = 110\ \Omega$$

we get the numerical solution

$$I_1 = 6.37\text{ mA},\ \ I_2 = 4.32\text{ mA}\ \text{ and }\ I_3 = -2.05\text{ mA}\,.$$

Note that for these resistance and voltage values, I_3 turns out to be negative.

In addition to calculating currents, we can use the calculated current values to calculate voltages between junctions. Define V_{12} to be voltage (potential difference) when moving from junction 1 to junction 2.

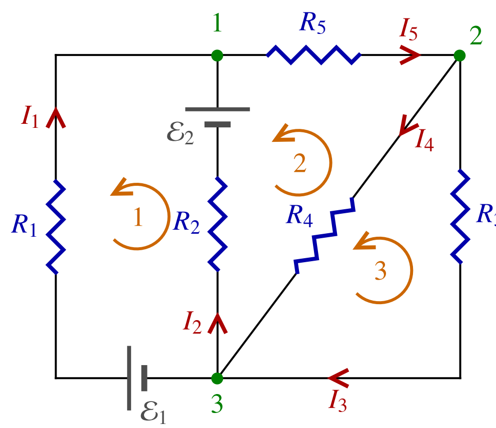
$$V_{12} = \Delta V = V_2 - V_1$$

Using the sign conventions discussed in the loop rule section we can get an expression for this and evaluate it in the Mathematica notebook.

$$V_{12} = \Delta V = -I_3 R_3 = 0.226\text{ V}$$

There are multiple paths you could take between these two junctions but because of the loop rule you would get the same value for any path.

- **A Three-loop Example**



There are five branches in this circuit and thus there are five currents; these have been labeled with an assigned direction. The three junctions have been labeled as have the three small loops. After omitting (any) one of the 3 junction rule equations, as we discussed earlier, we get two independent equations. The loop rule gives the other three independent linear equations we need to solve for the 5 currents.

$$I_1 + I_2 = I_5 \qquad \text{(Junction 1 Equation)}$$

$$I_5 = I_3 + I_4 \qquad \text{(Junction 2 Equation)}$$

$$I_3 + I_4 = I_1 + I_2 \qquad \text{(Junction 3 Equation - Omit this)}$$

The number of wires that meet at the junction must equal the number of currents in that junction rule equation; four wires meet at junction 3 and there are four currents in that equation. Here we omit the third equation.

For the three loops we have:

$$0 = \mathcal{E}_2 + I_1 R_1 - \mathcal{E}_1 - I_2 R_2 \qquad \text{(Loop 1 Equation)}$$

$$0 = -\mathcal{E}_2 + I_2 R_2 + I_4 R_4 + I_5 R_5 \qquad \text{(Loop 2 Equation)}$$

$$0 = -I_4 R_4 + I_3 R_3 \qquad \text{(Loop 3 Equation)}$$

The five remaining equations will be solved analytically in the Mathematica notebook. It is not instructive to write down the analytical solution. Using these values for the EMFs and resistances

$$\mathcal{E}_1 = 1.5\text{ V},\ \ \mathcal{E}_2 = 1.5\text{ V},\ \ R_1 = 200\ \Omega\,,\ \ R_2 = 400\ \Omega\,,\ \ R_3 = 110\ \Omega\,,\ \ R_4 = 240\ \Omega\ \text{ and }\ R_5 = 330\ \Omega$$

we get the numerical solution

$$I_1 = 1.86\text{ mA},\ \ I_2 = 0.928\text{ mA},\ \ \ I_3 = 1.91\text{ mA},\ \ I_4 = 0.875\text{ mA}\ \text{ and }\ I_5 = 2.278\text{ mA}$$

Looking at the circuit and using the loop rule sign conventions we can find expressions for V_{12}, V_{13} and V_{23} and then using the values in the Mathematica notebook find their numerical values.

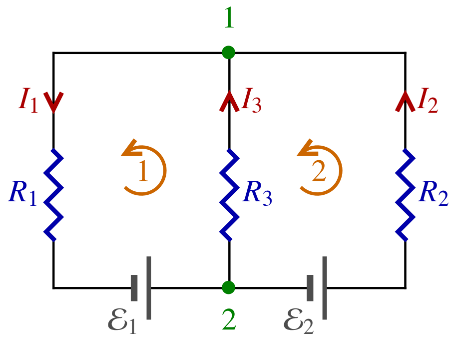
$$V_{12} = -I_5R_5 = -0.752\text{ V},\ \ \ V_{13} = -\mathcal{E}_2 + I_2R_2 = -1.13\text{ V}\ \text{ and }\ V_{23} = -I_4R_4 = -0.210\text{ V}$$

Procedure:

- There are three circuits you will solve. Parts A and B involve two-loop circuits and you will solve them analytically, numerically and experimentally. Part C is a three-loop circuit and you will solve it analytically and numerically but not by experiment.
- The numerical values will be listed as the theoretical values in the data tables.
- The experimental values will be measured using multimeters. Unlike the previous circuit experiments, here you must be careful to measure the signs of the currents and voltages correctly.
- To measure the current with the correct sign, follow the convention for the direction of the current and have the current enter the red terminal of the multimeter, set as an ammeter, and leave through the black terminal.
- The voltage between junctions will also be measured with attention to the sign. To measure V_{12} with the correct sign, connect the black terminal of the multimeter/voltmeter to junction 1 and the red terminal to 2.

Kirchhoff's Rules Data Sheet Name______________________ Group ______

Part (A)



1. Using the junction rule and loop rule, write down an independent set of equations to solve for the currents. Remember to omit one of the junction rule equations to get an independent set of equations.

2. Write down an expression for V_{12} in terms of the EMF and current variables.

3. Use the Mathematica notebook to symbolically solve this system of equations. Correctly input the correct equations, evaluate the solution and then write down the symbolic solutions for all three currents.

Set Up Circuit

Choose three resistors between 100 Ω and 1000 Ω. Record their resistance values as read from the stripes. Measure the source voltages of the EMFs. Set up the circuit shown above on the circuit board.

- Use these values for all theoretical calculations below.
- Resistance values are read from the stripes.
- Use the multimeter to read the voltage values.

	Resistance (Ω)
R_1	
R_2	
R_3	

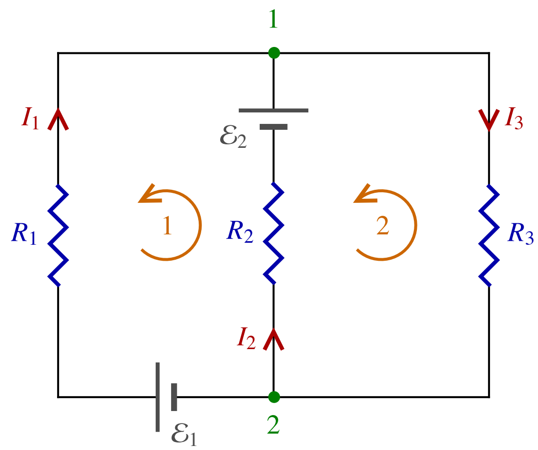
	dc Voltage - EMF (V)
$\mathcal{E}_1$	
$\mathcal{E}_2$	

Data Table

Following the discussion in the Procedure, measure the currents voltages paying attention to signs. Using the Mathematica notebook calculate the numerical (theoretical) values. Find the percent error between the experimental and theoretical values.

	Measured Value (from meter)		Theoretical Value (from Mathematica calculation)		Percent Error
I_1		A		A	
I_2		A		A	
I_3		A		A	
V_{12}		V		V	

Part (B)



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Choose three resistors between 100 Ω and 1000 Ω. Record their resistance values as read from the stripes. Measure the source voltages of the EMFs. Set up the circuit shown above on the circuit board.

- Use these values for all theoretical calculations below.
- Resistance values are read from the stripes.
- Use the multimeter to read the voltage values.

	Resistance (Ω)
R_1	
R_2	
R_3	

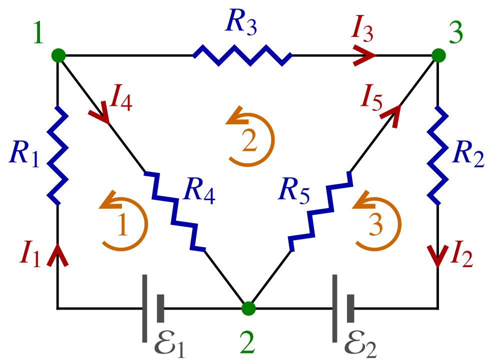
	dc Voltage - EMF (V)
$\mathcal{E}_1$	
$\mathcal{E}_2$	

Data Table

Following the discussion in the Procedure, measure the currents voltages paying attention to signs. Using the Mathematica notebook calculate the numerical (theoretical) values. Find the percent error between the experimental and theoretical values.

	Measured Value (from meter)		Theoretical Value (from Mathematica calculation)		Percent Error
I_1		A		A	
I_2		A		A	
I_3		A		A	
V_{12}		V		V	

Part (C)



1. Using the junction rule and loop rule, write down an independent set of equations to solve for the currents. Remember to omit one of the junction rule equations to get an independent set of equations.

2. Write down an expression for V_{12}, V_{13} and V_{23} in terms of the EMF and current variables.

3. Use the Mathematica notebook to symbolically solve this system of equations. Correctly input the correct equations, evaluate the solution. The symbolic solution is messy and there is no need to write it down.

Calculate Voltages and Currents

Choose five resistors between 100 Ω and 1000 Ω. Record their resistance values as read from the stripes. Measure the source voltages of the EMFs. Set up the circuit shown above on the circuit board.

- Use these values for all theoretical calculations below.

	Resistance (Ω)
R_1	500
R_2	300
R_3	200
R_4	800
R_5	250

	dc Voltage - EMF (V)
$\mathcal{E}_1$	1.5
$\mathcal{E}_2$	1.5

Data Table

Using the Mathematica notebook calculate the numerical (theoretical) values.

	Theoretical Value (from Mathematica calculation)	
I_1		A
I_2		A
I_3		A
I_4		A
I_5		A
V_{12}		V
V_{13}		V
V_{23}		V

Questions

1. For Part A, write down the loop rule equation for loop 1 if the loop is traversed clockwise instead of counterclockwise. Compare this with your loop 1 expression.

2. Show that in Part C, the voltages between junctions satisfy $V_{12} + V_{23} = V_{13}$. Explain why they should add like this.

3. Show that in Part C, the junction rule equation for junction 3 is the sum of the junction rule equations for junctions 1 and 2.

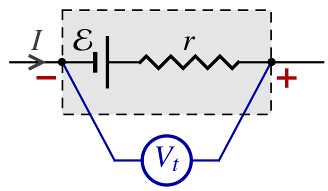
Internal Resistance and EMF of a Battery

Theory

An ideal DC voltage source has a fixed voltage, which doesn't change as you connect it to a circuit and draw a current from it. A real-world voltage source behaves as an ideal voltage source, its electromotive force (EMF), in series with a resistor, its internal resistance r. The voltage measured across the terminals of a DC source, the terminal voltage V_t, is smaller than its EMF when a current is drawn from the source.

$$V_t = \mathcal{E} - I\,r$$

In this experiment you will measure the internal resistance r and EMF $\mathcal{E}$ of a battery. In the diagram below the dashed line represents a battery. Inside that battery there is an ideal voltage source, labeled $\mathcal{E}$ and internal resistance labeled r. Across the terminals of the battery there is a voltmeter, measuring the terminal voltage.



The Design Experiment

This is an open-ended design experiment. Using the circuit boards and multimeters from the previous circuit experiments, you will design, perform and document an experiment to measure the both the internal resistance r and EMF $\mathcal{E}$ of a battery. The lab report must include:

- A clear description of the procedure you used.
- Sufficient data to make a graph that is used to find the results.
- The graph must include at least five data points, the best-fit line and the equation of that best-fit line.
- The values from the best-fit line must be used to find r and $\mathcal{E}$ and this must be clearly described.

Charged Particles in Electromagnetic Fields

Equipment and Setup: Mathematica file – CrossedFields.nb

Theory

Electric and Magnetic Forces

In the second part of the Electric Fields experiment we considered the motion of a charged particle shot in a region of a uniform electric field. The initial velocity $\vec{v}_0$ of the particle was in the *y*-direction, toward the top of the screen, with the value $\vec{v}_0 = v_0\hat{y}$ and the electric field was in the positive- or negative-*x* directions: $\vec{E} = E_x\hat{x}$, where the *x* direction was to the right of the screen. In this experiment we follow the same conventions for $\vec{v}_0$ and $\vec{E}$, but we will add a magnetic field that is into or out of the screen, $\vec{B} = B_z\hat{z}$, where the +*z*-direction is out of the screen. The magnitudes of the fields are found from the absolute values of the components: $E = |E_x|$ and $B = |B_z|$.

The electric force on a charge q is in an electric field $\vec{E}$ is given by

$$\vec{F}_{\text{elec}} = q\vec{E} \qquad \text{(Equation 1)}$$

The magnetic force on q in a magnetic field $\vec{B}$ depends on its velocity $\vec{v}$ and involves the vector product (cross product):

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B} \qquad \text{(Equation 2)}$$

The magnitude of the magnetic force is

$$F_{\text{mag}} = |q|\, v\, B \sin\theta \qquad \text{(Equation 3)}$$

where θ is the angle between the velocity vector and magnetic field vector. The direction of the vector product is found using the right hand rule. Note that the right hand rule is used to find the direction of the vector product $\vec{v} \times \vec{B}$ but if the charge is negative, the force is opposite to that direction.

Because of the nature of the vector product, the magnetic force is perpendicular to both the velocity $\vec{v}$ and the magnetic field $\vec{B}$. Any force perpendicular to the velocity does not affect the speed of a particle. It follows that the magnetic force, acting by itself, only changes the direction of a particle but not its speed.

The term electromagnetic field is often used when there are both electric and magnetic fields. When we have both fields, the force is the sum of the electric and magnetic terms. This combined expression is known as the Lorentz force law:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \text{(Equation 4)}$$

When we combine the Lorentz force law with Newton's second law, we get an expression for the acceleration of a charged particle in an electromagnetic field:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m\vec{a}$$

The acceleration determines the kinematics of a particle. In any electromagnetic field, two particles with the same charge to mass ratio q/m will have identical kinematics.

This experiment uses the same interface as the second part of the Electric Fields experiment but now we add the magnetic fields as well. We will use the same particles as in that experiment. For reference, the table of particle properties from that experiment is repeated below.

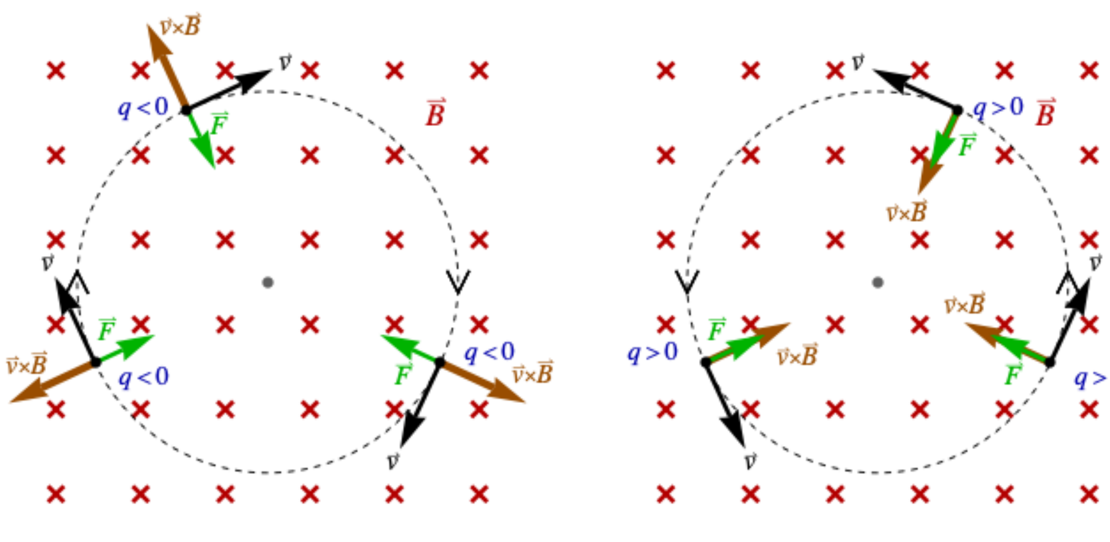
	Mass	Charge
electron: e^-	9.109×10^{-31}kg	$-e$
proton: p	1.673×10^{-27}kg	$+e$
neutron: n	1.675×10^{-27}kg	0
alpha particle: α	6.645×10^{-27}kg	$+2e$
positron: e^+	9.109×10^{-31}kg	$+e$

The charges are written in terms of the elementary charge e:

$$e = 1.602 \times 10^{-19}\text{C}$$

Motion of Charged Particles in a Uniform Magnetic Field

Suppose a particle has a velocity that is perpendicular to a uniform magnetic field, as shown in the figures below. The field will deflect the particle but, as discussed earlier, it will not affect the particle's speed. The magnetic force is perpendicular to the velocity of the particle and will stay perpendicular to the velocity as the particle moves. The effect of this is that the particle will trace out a circular trajectory with a constant speed. In Physics I, this was referred to as uniform circular motion.



Both figures show a charged particle moving in a uniform magnetic field directed into the page. On the left the particle is negatively charged and moves clockwise and on the right it is positively charged and moves counterclockwise. The force is always centripetal, toward the center.

In Physics I, you learned that when a particle moves in uniform circular motion, its acceleration is centripetal (toward the center) and it has the magnitude $a_c = v^2/r$, where v is the speed and r is the radius of the circle. The radius of the resulting circular trajectory shown above is

$$r = \frac{m\,v}{|q|\,B} \qquad \text{(Equation 5)}$$

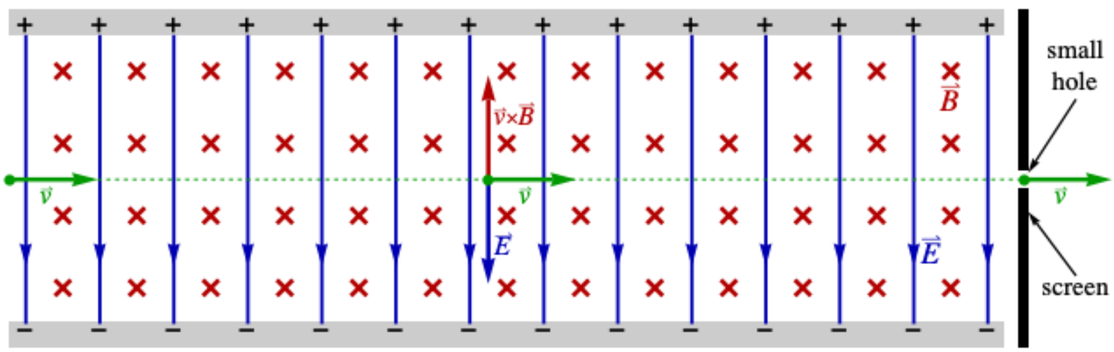
In a question at the end of the experiment, you will be asked to use Newton's second law to derive this result. In sections A through C you will consider the case where there is no electric field and just a uniform magnetic field. You will observe the circular trajectories.

Motion of Charged Particles in Uniform Crossed Fields

In sections D and E you will use both electric and magnetic fields. These fields are perpendicular to each other; we refer to this arrangement as *crossed fields*. An electric field in the $\pm x$-direction and a magnetic field in the $\pm z$-direction can only exert forces in the xy-plane. As a consequence, any particle with a velocity in the xy-plane will stay in that plane. Different values of the two fields can give some quite exotic particle trajectories.

The Velocity Filter

A velocity filter, also called a *velocity selector*, is a device that takes a beam of different particles (with different charges and masses) and allows particles with one specific speed to exit. The diagram below shows a velocity filter.



In this arrangement, the electric field, magnetic field and velocity are mutually perpendicular. To select for particles with the precise velocity $\vec{v}$ we must arrange the fields so that the electric and magnetic forces cancel.

In the figure above, a beam of particles is shot into an arrangement with crossed fields. The particle enters on the left and electric and magnetic forces act on it as it moves. There is a screen with a small hole on the far right of the setup and only the particles that are undeflected will pass through the hole and exit. From the Lorentz force law (Equation 4) we can see that given some velocity $\vec{v}$, if we choose the electric and magnetic fields to satisfy $\vec{E} = -\vec{v} \times \vec{B}$, then only particles with that exact velocity will not be deflected:

$$\vec{E} = -\vec{v} \times \vec{B} \quad \Longrightarrow \quad \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = \vec{0}$$

Charged Particles – Worksheet Name________________ Group_____

Computer Setup

- The interactive panel shows the paths of various charged particles that are shot into a region of uniform electric field and uniform magnetic field. The electric field points left-right; a positive value of E_x corresponds to a field to the right. The magnetic field points *out of* the screen or *into* the screen; a positive value of B_z corresponds to a magnetic field pointing out of the screen. The particle is shot in the positive *y*-direction with initial speed v_0.
- At the top right are two buttons, a reset button and a U-shaped update button. The left of the control panel allows for the selection of the particle. The choices are: electron, proton, neutron, alpha particle and positron.
- In the middle of the control panel you can choose the value of the initial speed v_0, the electric field E_x, and the magnetic field B_z. Anytime you change these values, you should then click the update button at the upper right. Note that the magnitudes of the fields *B* and *E* are the absolute values of B_z and E_x.
- In the middle there is also a checkbox to Animate Motion. Checking this shows controls for animation. If you get a dialog box that says "Dynamic Content Warning", click "Enable Dynamic". Enabling animation may slow things down too much; if so, uncheck the checkbox.
- The Exit Data is listed below the control panel.

Section A: Electron in Uniform Magnetic Field

1. Use $v_0 = 600{,}000$ m/s, $E_x = 0$ and $B_z = -0.00015$ T. Select the electron e^- and record the exit data.

x_f = ______________ cm, y_f = ______________ cm, t_f = ______________ ns

Questions

A-1. What is the direction of the magnetic force on the electron as it enters the field?

A-2. Calculate the radius of the circular path from the initial data and compare with the exit data. **Show your work in the space below.**

A-3. Which way (*into* the screen or *out of* the screen) should the magnetic field be directed in order to make the electron go *counterclockwise*? (Circle your answer.)

a) into the screen

b) out of the screen

A-4. Referring to (Equation 5), calculate the magnitude of the magnetic field (in teslas) needed to cause the electron with $v_0 = 600{,}000\,\text{m/s}$ to move in a counterclockwise circular path of radius 4 cm. **Show your work in the space below.**

Section A: Electron in Uniform Magnetic Field (cont'd)

2. We want the electron to move in a counterclockwise circle of radius 4 cm. Use the results of Questions A-3 and A-4 to do this, remembering that the sign of B_z gives the direction of the *B*-field. If the radius of the path is not 4 cm, review your calculation in Question A-4, if necessary, to correct any errors. Record the exit data.

x_f = ______________ cm, y_f = ______________ cm, t_f = ______________ ns

Section B: Positron in Uniform Magnetic Field

1. Now keep all values set as they are (after A.2) but select the positron e^+ instead of the electron. The positron is the antiparticle of the electron. Record the exit data.

$x_f =$ ______________ cm, $y_f =$ ______________ cm, $t_f =$ ______________ ns

Questions

B-1. Describe and explain the similarities and differences between the electron and positron paths.

Section C: Alpha Particle in Uniform Magnetic Field

1. Use $v_0 = 600{,}000\text{ m/s}$, $E_x = 0$ and $B_z = -0.00015\text{ T}$. Select the alpha particle α and record the exit data.

$x_f =$ ______________ cm, $y_f =$ ______________ cm, $t_f =$ ______________ ns

2. Section C is similar to Section A. This time, though, the particle is an *alpha particle* (a helium nucleus). You are to use a magnetic field to cause the alpha particles to move clockwise in a semicircular path of approximately 4-cm radius. You should **not** use an electric field. Before proceeding, answer Question C-1 below.

Questions

C-1. Which way (into the screen or out of the screen) should the magnetic field be directed? (Circle your answer.)

a) into the screen

b) out of the screen

Section C: Alpha Particle in Uniform Magnetic Field (cont'd)

3. Now experiment with changing the magnetic field until you get the alpha particles to move clockwise in a semicircle of 4-cm radius. When you are successful, record your value of the magnetic field magnitude B in teslas in the space below.

$B =$ __________________ T

Section D: Electric and Magnetic Fields I

1. Use $v_0 = 600,000$ m/s, $E_x = 20$ N/C and $B_z = -0.00015$ T. Select the positron e^+. With an electric field in the *x*-direction and magnetic field in the *z*-direction a charged particle moving in the *x-y* plane will stay in that plane. With this choice of parameters an odd motion is observed. **Sketch the trajectory in the space below.**

Questions

D-1. Consider two positions in the trajectory: a point we will refer to as **L** at the far left, where the radius of curvature is the smallest, and a point **R** at far right where the radius of curvature is largest. Draw these points and label them in the above diagram. For each of these points, draw vectors showing the directions of the electric and magnetic forces $\vec{F}_{elec}$ and $\vec{F}_{mag}$.

D-2. Of the two positions **L** and **R**, where is the speed the largest? **Explain. Hint:** Consider the electric potential energy and remember that electric field lines point toward lower potential. Also remember that magnetic forces do no work and therefore cannot change the speed. **Answer in the space below.**

Section E: Electric and Magnetic Fields II – The Velocity Filter

1. Here we will set up a velocity filter as discussed in the theory section. Use $v_0 = 600,000$ m/s, $B_z = 0.0002$ T, and select the electron e^-.

Questions

E-1. Which direction (left or right) should the electric field point in order for the velocity filter to work properly? (Circle your answer.)

a) to the left

b) to the right

E-2. Given the above values of *v* and *B*, calculate the value *E* required for particles to go through without being deflected. **Show work in the space below.**

Section E: Electric and Magnetic Fields II – The Velocity Filter (cont'd)

2. Input the appropriate value of E_x, being careful about the sign.

3. Record *x* from the Exit Data for the electron.

Exit Data: e^-: x_f = _________ cm

4. Now try the rest of the particles (proton, neutron, alpha particle and positron) and record their exit values.

Exit Data:

p: x_f = _________ cm

n: x_f = _________ cm

α: x_f = _________ cm

e^+: x_f = _________ cm

Additional Questions (can be answered outside of class)

1. In the space below, give a derivation of (Equation 5). **Hint:** Apply Newton's second law to the centripetal direction.

2. In Section A, what happens to the speed of an electron as it moves in its semicircular path? **Explain.**

3. In Section C part 2, do you need a *larger* or *smaller* B-field magnitude than in Section A part 4? **Explain.**

4. Calculate the B-field needed in Section C and compare this with the value of B you found in Step 3 of Section C. Calculate the percent error. **Show your work in the space below.**

Name __________________ Group ______

Induction – Magnet Through a Coil

Equipment and Setup: Voltage probe, Solenoid, Bar magnets (2), Capstone file – Induction.cap

Background

When a magnet is passed through a coil there is a changing magnetic flux through the coil. This induces an *emf* (electro-motive force) ε in the coil. According to Faraday's Law of Induction:

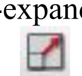
$$\mathcal{E} = -N\frac{d\Phi}{dt}.$$

Use the voltage sensor to measure the voltage (*emf*) induced in a solenoid as a bar magnet moves through the solenoid. A plot of the voltage versus time is made and the area under the curve is found by integration. This area (ε×*t*) is proportional to the total magnetic flux ΔΦ that passes through the coil during that time, since:

$$\int \mathcal{E}\, dt = -N\,\Delta\Phi.$$

Setup and Data Recording

auto-expand tool



selection tool

area tool

1. Connect the red lead of the voltage probe to the top terminal of the solenoid and the black to the bottom. A positive voltage corresponds to the top terminal being at higher voltage.
2. Hold the magnet so that the north end is a few centimeters above the solenoid.
3. Start recording (press rec) just before dropping the magnet through solenoid. Stop recording after the magnet falls through. It will shut off automatically after 5 seconds but stopping earlier will help.
4. The data from the voltage sensor should be recorded automatically on the graph.
5. Click the selection tool and reposition and resize it to cover one of the peaks. Click the area tool and record the area. Do the same for the other peak.
6. Ignoring the signs, calculate the percent differences for the two areas.
7. Sketch the shape of your graph in the space to the right of the data. Pay close attention to any similarities or differences in the two peaks.
8. Repeat this procedure for each case listed in the data sheet.

Data Sheet

A. Look at the way the coil is wrapped. If a current passes through the solenoid from the bottom to the top, is the current clockwise or counterclockwise when viewed from above? The sense of an *emf* (clockwise or counterclockwise) corresponds to the sense of its induced current. An *emf* pushing from the bottom to the top will record as a positive voltage.

B. Drop one magnet through the solenoid with the north pole on the bottom. Record the value of integration for first peak, the value of integration for the second peak and the percent difference. To the right of the table give a crude sketch of the graph that is shown.

	First Peak (V·s)	Second Peak (V·s)	% Difference
Area			
Clockwise or Counterclockwise			

C. Drop one magnet through the solenoid with the south pole on the bottom. Record the value of integration for first peak, the value of integration for the second peak and the percent difference. To the right of the table give a crude sketch of the graph that is shown.

	First Peak (V·s)	Second Peak (V·s)	% Difference
Area			
Clockwise or Counterclockwise			

D. Drop two magnets through the solenoid taped together so that both south poles are together and at the bottom. Record the value of integration for first peak, the value of integration for the second peak and the percent difference. To the right of the table give a crude sketch of the graph that is shown.

	First Peak (V·s)	Second Peak (V·s)	% Difference
Area			
Clockwise or Counterclockwise			

Questions

1. Is the incoming (first peak area) flux equal in magnitude to the outgoing (second peak area) flux? Should they be equal? Explain.

2. The two peaks do not have exactly the same shape. Explain how they are different and why?

3. Why do the peaks have opposite signs?

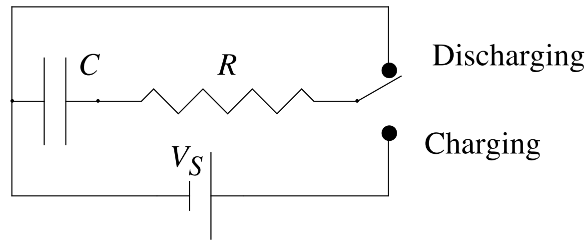
4. Use Lenz's law to theoretically explain the sense of the induced *emf* as a magnet with its North pole down as the magnetic first enters and then leaves the solenoid.

5. Suppose an isolated magnetic North pole is discovered and dropped through this setup. Describe the voltage pattern by giving a crude sketch of the voltage as a function of time.

RC Circuits

Equipment and Setup: RC circuit board, Voltage probe, Capstone file – RC circuits.cap

We will study charging and discharging capacitors. The characteristic time for charging or discharging a capacitor is called the time constant τ: $\tau = RC$. The charge on the capacitor and the voltage across it are related by $V = Q/C$. A fully charged capacitor will have charge $Q_{max} = CV_S$. The charge and voltage for charging and discharging capacitors are given by:



$$Q(t) = Q_{\max}\left(1 - e^{-t/\tau}\right) \text{ and } V(t) = V_S\left(1 - e^{-t/\tau}\right) \text{ for charging}$$

$$Q(t) = Q_{\max}\, e^{-t/\tau} \quad \text{ and } \quad V(t) = V_S\, e^{-t/\tau} \quad \text{ for discharging}$$

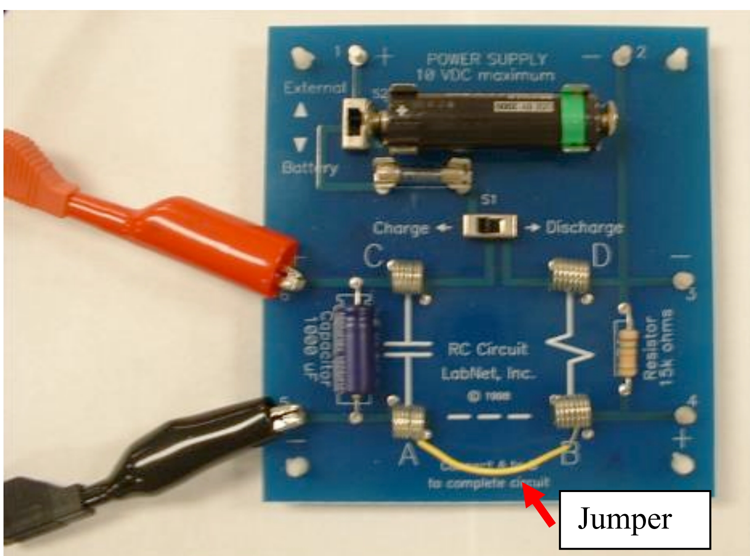
where we are assuming that the capacitor is fully charged before it is discharged and fully discharged before it is charged. (Hint: To fully discharge the capacitor, set the switch to discharge and move the jumper to A-C to short the capacitor, then back to A-B. To fully charge the capacitor, set the switch to charge and move the jumper to A-D to short the resistor. This reduces the time constant and thus speeds up the process.)

Procedure

(1) Connect the voltage probe to Analog Channel A of the Pasco Interface. Open the Physics Folder and Open the Capstone file called RC Circuit.

(2) On the RC circuit board, ensure the switch to the left of the battery is set to 'Battery.'

(3) Measure the battery voltage: Clip the alligator clips of the voltage probe directly across the battery and click record for a second, then click 'stop recording.' Record the battery voltage on the Data sheet.



(4) Set the switch below the battery to 'discharge.' Connect the voltage probe across the capacitor. Briefly connect the jumper wire across A-C to short the capacitor, then connect it to A-B to complete the RC circuit.

(5) Calculate the theoretical value of the time constant, $\tau = RC$, in standard SI units.

(6) Charge the capacitor: Switch the switch to charge and click the record button on the capstone program. Record for about one minute. Record 12 of the voltage values on your data sheet. Choose times that illustrate the proper functional behavior; include a few time constants in your data.

(7) Using Software, plot a graph of $\ln(V_S - V)$ vs. time. Include the best-fit line and its equation.

(8) From the slope find the experimental value of the time constant. In theory the slope should be $-1/\tau$, so the experimental value of the time constant is: $\tau_{\text{exp}} = -1/slope$.

(9) Fully charge the capacitor: leave the switch in charge and briefly connect the jumper to A-D, then switch it back to the normal position A-B.

(10) Discharge the capacitor: Switch to switch to discharge and click the record button on the capstone program. Record for about one minute. Record 12 of the voltage values on your data sheet. As before choose times that illustrate the proper functional behavior.

(11) Using Software, plot a graph of $\ln V$ vs. time. Include the best-fit line and its equation.

(12) From the slope find the experimental value of the time constant. In theory the slope should be $-1/\tau$, so the experimental value of the time constant is: $\tau_{\text{exp}} = -1/slope$.

(13) Average the two experimental values of the time constant (one from charging and one from discharging) and compare with the theoretical value. Give the percent error.

(14) Place a second capacitor in parallel to the first. (The position is marked on your board.) The new capacitance is the sum of the two. Repeat steps (4) through (14) with the new time constant.

(15) When you finish, please set the switch to 'discharge,' and remove the jumper wire and second capacitor. Put the RC circuit card, jumper, and extra capacitor into the same storage bag.

RC Circuits Data Sheet Name ________________________ Group ______

Source Voltage = V_S = ___________________________

<table>
<tr><th colspan="4">Trial 1</th><th colspan="4">Trial 2</th></tr>
<tr><td colspan="4">R = _______________ C = _______________
τ = R C = ________________________</td><td colspan="4">R = _______________ C = _______________
τ = R C = ________________________</td></tr>
<tr><td colspan="2">Charging</td><td colspan="2">Discharging</td><td colspan="2">Charging</td><td colspan="2">Discharging</td></tr>
<tr><td>Time</td><td>Voltage</td><td>Time</td><td>Voltage</td><td>Time</td><td>Voltage</td><td>Time</td><td>Voltage</td></tr>
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<tr><td colspan="2">Slope = __________
τ_{exp} = __________</td><td colspan="2">Slope = __________
τ_{exp} = __________</td><td colspan="2">Slope = __________
τ_{exp} = __________</td><td colspan="2">Slope = __________
τ_{exp} = __________</td></tr>
<tr><td colspan="4">Average τ_{exp} = __________________
% Error = __________________</td><td colspan="4">Average τ_{exp} = __________________
% Error = __________________</td></tr>
</table>

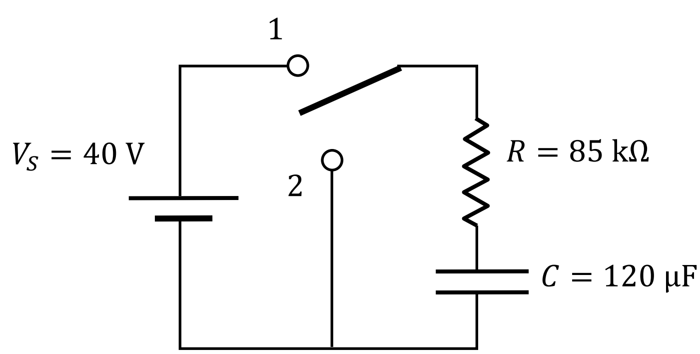
Remember to turn in all 4 graphs. Charging graphs: $\ln(V_S - V)$ vs. time. Discharging graphs: $\ln V$ vs. time. (See instructions on previous page.)

Questions:

1. Starting with the equation for charging $V(t) = V_S\left(1 - e^{-t/\tau}\right)$, derive the linearized equation $\ln(V_S - V) = -\left(\frac{1}{\tau}\right) t + \ln V_S$. What is the significance of the slope of this equation? (Note: we can't take the natural log of something with units, so presumably we divided V and V_S by one volt prior to graphing.)

2. Starting with the equation for discharging $V(t) = V_S\, e^{-t/\tau}$, derive the linearized equation $\ln V = -\left(\frac{1}{\tau}\right) t + \ln V_S$. What is the significance of the slope of this equation?

3. For the series RC circuit shown, the capacitor is initially discharged (connected to position 2 for a long time). The switch is changed to position 1 and the capacitor is allowed to charge for 8.00 s. After that the switch is quickly switched back to position 2 and the capacitor is allowed to discharge. After discharging for 6.00 s, what is the voltage across the capacitor, V.



Series RLC Circuit: Impedance and Phase

Equipment and Setup: RCL circuit board, Multimeter, Banana leads (one black, one red), Voltage probe, Capstone file – RLC Impedance.cap

Theory

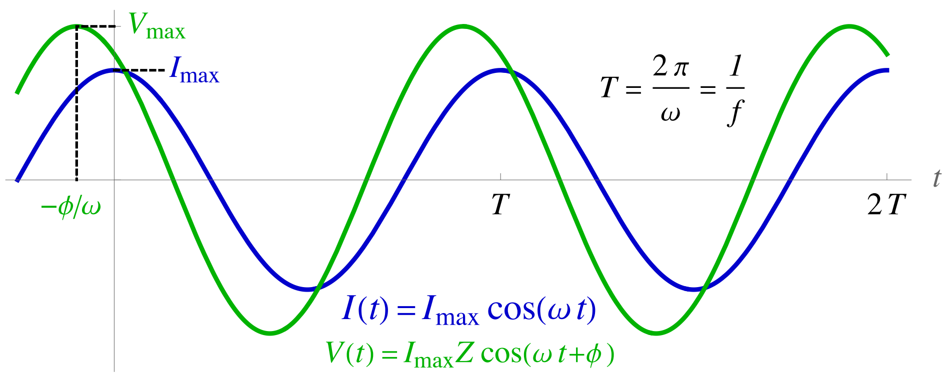
General AC Circuits

An AC (alternating current) source produces a voltage that is a sinusoidal function of time. The general form of a sinusoidal function (in this case a function for voltage) is $V(t) = V_{\max}\cos(\omega t + \phi)$. $V_{\max}$ is the *peak voltage,* or *voltage amplitude*. This is related to the rms (root-mean-square) voltage V_{rms} by $V_{\text{rms}} = \frac{1}{\sqrt{2}}V_{\max}$. The *angular frequency* ω is related to the *frequency* f by $\omega = 2\pi f$. The *phase angle* ϕ shifts the graph of voltage versus time along the time axis. We use the same notation for sinusoidal functions of current, replacing the V with I.

In any general AC circuit consisting of resistors, capacitors and inductors connected across some AC source, the voltage of the source and the current produced by the source are related by two values, the impedance Z and the phase ϕ. Here the phase angle is the relative phase between the voltage and current. The absolute phase is unimportant and can be removed by shifting the time variable. Choosing the absolute phase to be zero for the current we have

$$I(t) = I_{\max}\cos(\omega t) \quad \text{and} \quad V(t) = I_{\max} Z\cos(\omega t + \phi)$$

Plotting the voltage and current functions on the same graph gives:



The peak voltage and peak current are related by the impedance $V_{\max} = I_{\max}Z$. Note that the rms values are related similarly: $V_{\text{rms}} = I_{\text{rms}}Z$. For the phase, the voltage is ahead of the current by the phase angle. This means that for a positive phase angle, as shown in the figure above, the peak of the voltage occurs at an earlier time than the peak of the current. To average a periodic function, we integrate the function over one period and divide by the period. We can find an expression for the average power in a general AC circuit.

$$\mathcal{P}_{\text{ave}} = \langle V(t)I(t)\rangle_{\text{ave}} = \frac{1}{T}\int_0^T V(t)I(t)\,dt = V_{\max}\,I_{\max}\,\frac{1}{T}\int_0^T \cos(\omega t)\cos(\omega t + \phi)\,dt$$

Using the trig identity for the sum of angles: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ and integrating gives.

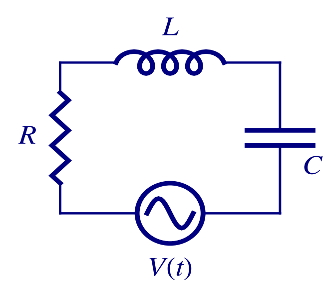
$$\mathcal{P}_{\text{ave}} = \frac{1}{2}V_{\max}\,I_{\max}\,\cos\phi = V_{\text{rms}}\,I_{\text{rms}}\,\cos\phi$$

For each of our three linear circuit elements, resistors, inductors and capacitors, the voltage across each element is related to the current through it. The voltage to current relations will give the impedance and phase angle for each of the three components taken by themselves. Note that the sine and cosine functions are the same functions shifted by $\pm\frac{\pi}{2}$: $\cos\left(\theta+\frac{\pi}{2}\right)=-\sin\theta$ and $\cos\left(\theta-\frac{\pi}{2}\right)=\sin\theta$.

	Voltage to Current Relation	$I(t)=I_{\max}\cos(\omega t)$ $V(t)=I_{\max}Z\cos(\omega t+\phi)$	Impedance Z	Phase angle ϕ
Just R	$V=IR$	$V(t)=IR=I_{\max}\cos(\omega t)\,R$ $=I_{\max}R\cos(\omega t+0)$	R	0
Just L	$V=L\dfrac{dI}{dt}$	$V(t)=L\dfrac{dI}{dt}=L\,(-I_{\max}\,\omega\sin(\omega t))$ $=I_{\max}\,\omega L\cos\left(\omega t+\dfrac{\pi}{2}\right)$	$X_L=\omega L$ $=2\pi fL$	$\dfrac{\pi}{2}=90°$
Just C	$V=\dfrac{q}{C}$ where $\dfrac{dq}{dt}=I$	$\dfrac{dq}{dt}=I\implies q(t)=I_{\max}\dfrac{1}{\omega}\sin(\omega t)$ $V(t)=\frac{q}{C}=I_{\max}\frac{1}{\omega C}\cos\left(\omega t-\frac{\pi}{2}\right)$	$X_C=\dfrac{1}{\omega C}$ $=\dfrac{1}{2\pi fC}$	$-\dfrac{\pi}{2}=-90°$

X_L and X_C are referred to as the inductive reactance and capacitive reactance.

Series RLC Circuits



A series RLC circuit has a resistor, inductor, and capacitor connected in series across an AC voltage source. Use the same phase convention as above, where the current is $I(t)=I_{\max}\cos(\omega t)$. Because we have a series circuit, this current is the same through all three circuit elements and the source, and the voltages at every instant add.

$$V(t)=V_R(t)+V_L(t)+V_C(t)$$

With the results from the table above, we can rewrite the expression for the voltage sum as:

$$I_{\max}Z\cos(\omega t+\phi)=I_{\max}R\cos(\omega t)+I_{\max}X_L\cos\left(\omega t+\frac{\pi}{2}\right)+I_{\max}X_C\cos\left(\omega t-\frac{\pi}{2}\right)$$

Using trig identities and rearranging gives

$$I_{\max}\left[Z\cos\phi\cos(\omega t)-Z\sin\phi\sin(\omega t)\right]=I_{\max}\left[R\cos(\omega t)-(X_L-X_C)\sin(\omega t)\right]$$

For this to be true at all times then the factors multiplying $\cos(\omega t)$ and $\sin(\omega t)$ on each side of the equation must be separately equal. This gives $Z\cos\phi=R$ and $Z\sin\phi=X_L-X_C$. Solving the circuit consists of solving for the impedance and phase. For our series circuit we get

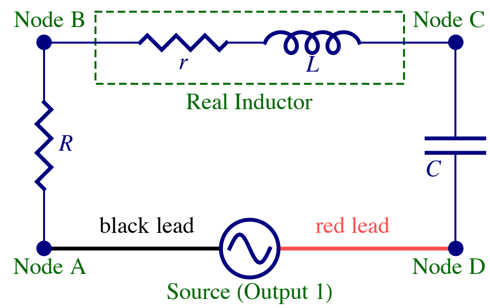
$$Z=\sqrt{R^2+(X_L-X_C)^2}\quad\text{and}\quad\tan\phi=\frac{X_L-X_C}{R}$$

Using $Z\cos\phi=R$ and $V_{\max}=I_{\max}Z$, the general expressions for the average power can be simplified for series circuits.

$$\mathcal{P}_{\text{ave}}=\frac{1}{2}V_{\max}I_{\max}\cos\phi=V_{\text{rms}}I_{\text{rms}}\cos\phi\quad\Longrightarrow\quad\mathcal{P}_{\text{ave}}=\frac{1}{2}I_{\max}^2R=I_{\text{rms}}^2R$$

We may view general series AC circuits as special cases of the series RCL circuit. If there is no resistor then set $R = 0$ and if no inductor or capacitor, then set X_L or X_C to zero. If there are multiple resistors, inductors or capacitors, then use the series equivalent values for R, C or L.

RLC Series Circuit with a *Real* Inductor



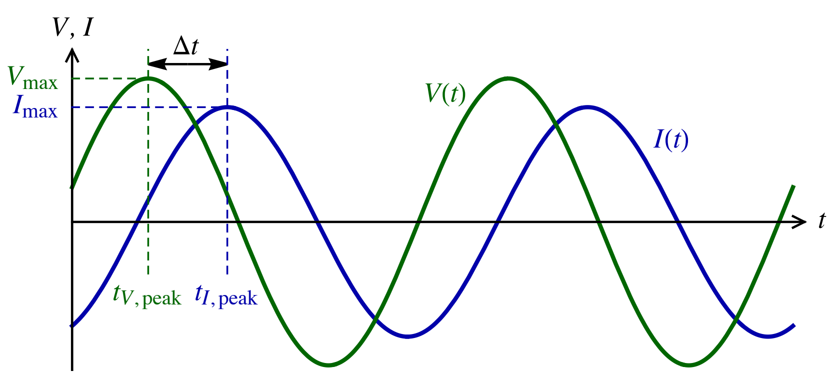
In the preceding analysis we assumed that the inductor, L, was ideal, that is, that it had no resistance. However, in this experiment our inductor is made of a coil of wire and has resistance as well as inductance. We will represent this in the diagram by an ideal resistor r in series with an ideal inductor L. These components are not actually separate. The impedance of our real inductor is *not* X_L. Since the only resistance is r and, as we discussed above, set $X_C = 0$, since there is no capacitor. The result is that the impedance of the real inductor is $Z_{rL} = \sqrt{r^2 + X_L^2}$. The peak voltage across the inductor is: $V_{rL,\max} = I_{\max} Z_{rL}$.

To find the impedance across the source use the equivalent resistance:

$$Z = \sqrt{R_{eq}^2 + (X_L - X_C)^2}\,, \text{ where } R_{eq} = r + R$$

Dual Trace Oscilloscope and Phase

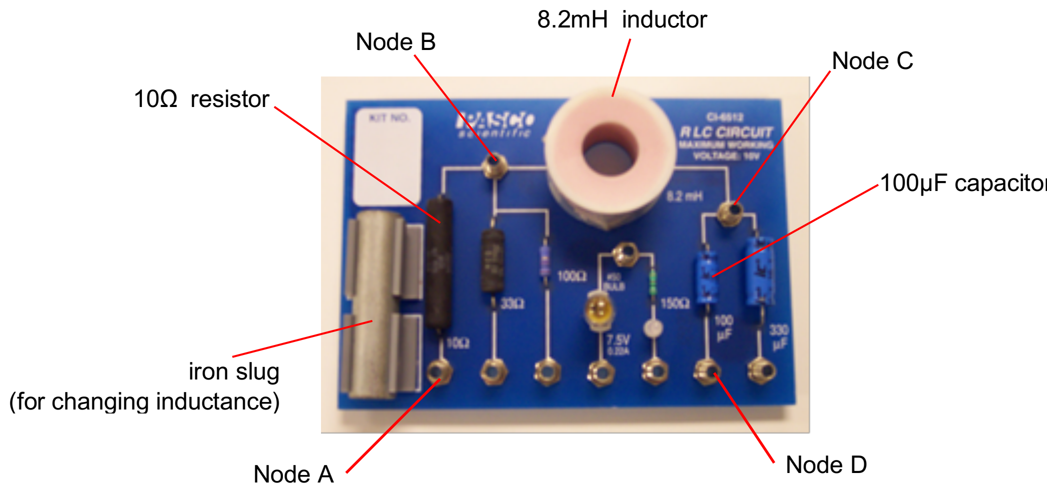
The Capstone software has an oscilloscope display. We will use this with its dual trace feature. This shows both current and voltage as functions of time. Using this display we will find the phase angle.



From the oscilloscope display we can read off the times corresponding to the peak values of voltage and current: $t_{V,\text{peak}}$ and $t_{I,\text{peak}}$. From this find Δt and then the phase angle.

$$\Delta t = t_{I,\text{peak}} - t_{V,\text{peak}} \quad \text{and} \quad \phi = \omega\,\Delta t$$

Procedure



Preliminaries:

On the RLC circuit card identify the following components:

$R = 10\ \Omega$ $L = 8.2\ \text{mH}$ $C = 100\ \mu\text{F}$

Part A. Voltage, Current, Impedance and Phase

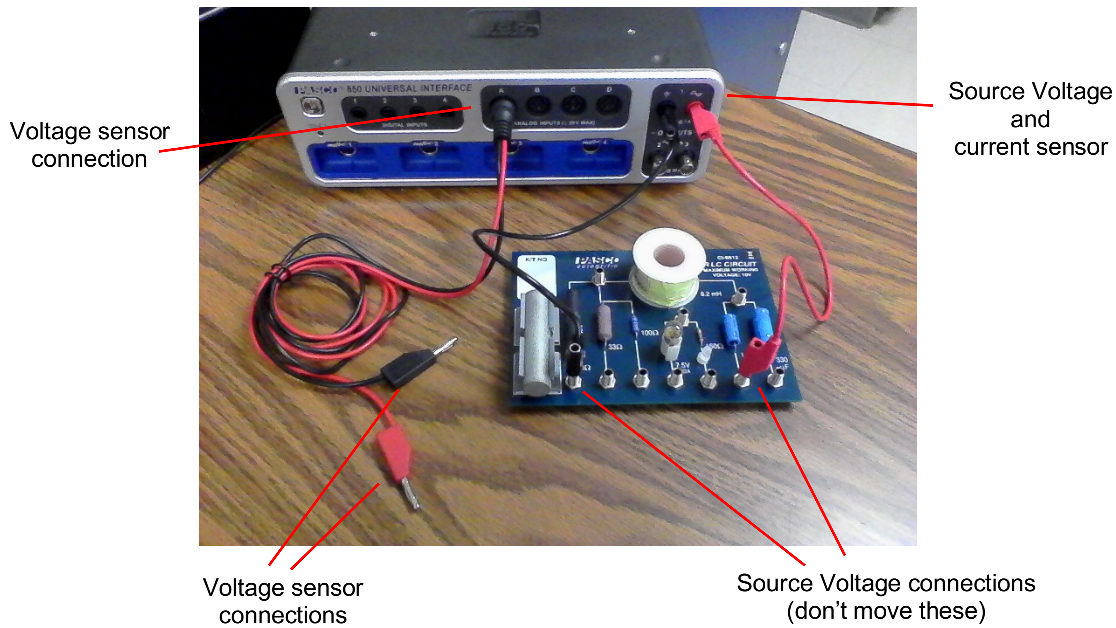
(A.1) Use a multimeter with banana leads to measure the internal resistance, *r*, of the inductor. This is the resistance between Node B and Node C. Record this value on the Data Sheet. Put the multimeter away, you won't need it again.

(A.2) Turn on the Pasco interface and open the Capstone file on your computer.

(A.3) Using the banana leads, connect output 1 on the Pasco interface (upper far right on Pasco interface) across the chosen RLC circuit. Connect the black banana lead from ground on the interface to Node A on the RLC circuit card. Connect the red banana lead from red output to Node D on the RLC circuit card. This is your source voltage. The ammeter is also contained in the Pasco interface. You will leave these leads connected throughout the rest of the experiment. **Don't move them.**

(A.4) Connect the **voltage probe** to Analog Input A on the interface. This is your voltmeter. You will connect these leads across various locations on the RLC circuit card. Note: the direction in which you connect these leads is very important. Pay close attention to the instructions.

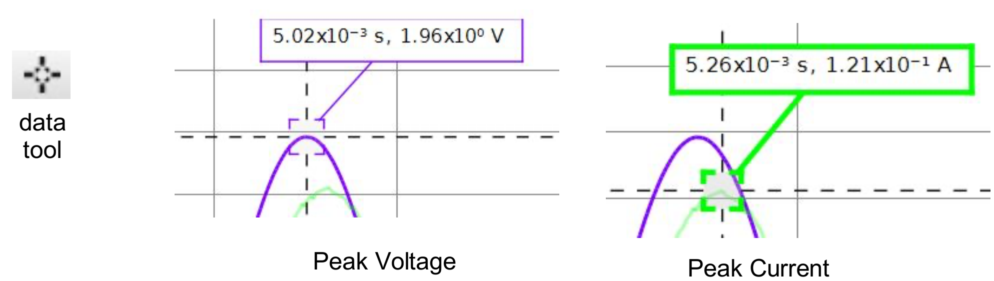
Your set up should resemble this.



(A.5) **SOURCE VOLTAGE**: Connect the voltage probe leads across the source voltage. Plug the red lead to Node D (plug it into the back of the red source lead). Plug the black lead into Node A (into the back of the black source lead).

(A.6) On the Capstone display, click Signal Generator (on left). Change the frequency to 120 Hz. Click "**on**" to turn the generator on. Click "Monitor" and monitor the signal for a few seconds.

(A.7) On the dual trace oscilloscope display (onscreen graph), identify the voltage trace and the current trace. Move the Data Selection tool (box with cross-hairs) on your display to measure the peak voltage (amplitude) of the source. Record this as the peak source voltage, V_{max} and the time of that peak $t_{V,\text{peak}}$ in Table A on the data sheet. Also, measure the peak current and time, and record this as *experimental* peak current I_{max} and also record $t_{I,\text{peak}}$. in Table A on your data sheet. The times are also shown; record these also in Table A as $t_{V,\text{max}}$ and $t_{I,\text{max}}$.



Qualitative phase analysis: Look at the two peaks, which peaks first in time, voltage or current? If voltage peaks first, then voltage *leads* current. If voltage peaks after current, then voltage *lags* current. If they peak at the same time then they are *in phase*. Record this in section B of the Data sheet.

$\frac{\pi}{2}$ radians is one quarter of a cycle. Look at the peaks. Is the difference in the peaks about $\frac{\pi}{2}$ or noticeably less than $\frac{\pi}{2}$? Record this in section A of the data sheet.

Record the time for the peak values of both the current and the voltage. Find $\Delta t = t_{I,\max} - t_{V,\max}$ and from that find the phase angle.

(A.8) **RESISTOR**: Leaving the source voltage connected as is, move the voltage probe leads so you can measure the voltage across the resistor alone. Leave the black lead connected to Node A. Move the red lead to B. Monitor for a few seconds.

Measure the peak voltage and record this in Table A as the *experimental* peak voltage across the resistor, $V_{R,\max}$. Does $V_{R,\max}$ *lead* or *lag* current, or are they *in phase*? Record your answer in section B of the data sheet.

(A.9) **INDUCTOR**: Leaving the source voltage connected as is, move the voltage probe leads to measure the voltage across the inductor alone. Move the black lead to Node B. Move the red lead to Node C. Monitor for a few seconds.

Measure the peak voltage. Record this in Table A as *experimental* peak voltage across the inductor, $V_{L,\max}$. Does $V_{L,\max}$ *lead* or *lag* current? By $\frac{\pi}{2}$ or less than $\frac{\pi}{2}$? Record your answers in section A of the data sheet.

(A.10) **CAPACITOR**: Leaving the source voltage connected as is, move the voltage probe leads so you can measure the voltage across the capacitor alone. Move the black lead to Node C. Connect the red lead to Node D (into the back of the red lead from the source). Monitor for a few seconds. Measure the peak voltage and record this as the *experimental* peak voltage across the capacitor, $V_{max,C}$. Does $V_{max,C}$ *lead* or *lag* current? By $\frac{\pi}{2}$ or less than $\frac{\pi}{2}$? Record your answers in section A of the data sheet.

Continue on to Part B. Later, you will compute theoretical values of these voltages and currents and compare them to your experimental values, and you can then answer the questions listed for section A. But don't do that now, finish getting experimental data first.

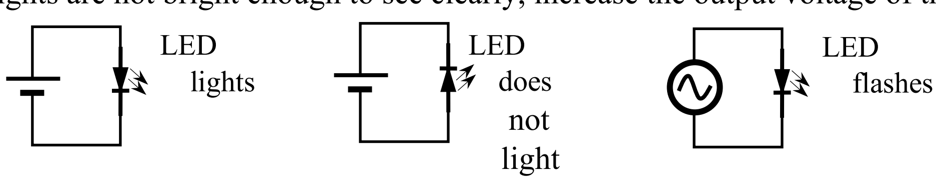
Part B. Changing the Inductance

(B.1) Unclip the iron core from your RLC circuit card and insert it into the center of the inductor. This will change the inductance to an unknown value. Move your voltage probe to measure the voltage across the inductor. Place the black lead on Node B and the red lead on Node C. Monitor the system for a few seconds.

(B.2) On the Capstone display, measure the peak current and record this as $I_{\max}$ in section B of your Data sheet. Measure the peak voltage and record this as $V_{L,\max}$ in section B on your data sheet.

Part C. Alternating Current, frequency, and Diodes.

A diode is a device that has a low resistance when current runs one way but a high resistance when current runs the other direction (reverse bias). An LED is a light emitting diode. It light when properly biased, but does not light when reversed biased. If connected to an AC source, the LED should blink at the frequency of the source. The LED housing on our RLC circuit card is actually a single LED, it combines two components in one housing. If the lights are not bright enough to see clearly, increase the output voltage of the source.



Note: If you have difficulty seeing the results due to reduced color vision, please feel free seek help in describing the results.

(C.1) Identify the LED on the RLC circuit board. Connect the source voltage across the LED (and the 150 Ω resistor). On the signal generator portion of Capstone display, set the source voltage to 5V and lower the frequency to about 5 Hz. Turn on the signal generator and look at the LED from above. On your data sheet describe what you see.

(C.2) Increase the frequency. On your data sheet, describe what you see.

(C.3) Increase the frequency until the light appears to be steady (no longer blinking). On your data sheet, describe what you see.

When you put the equipment away, please put the circuit cards, banana leads, and voltage probes away **separtely**. These do not go in the same bag!

Proceed to your calculations. Turn in only your data sheet and calculations. Do not include these instructions with your lab report.

Impedance and Phase – Data Sheet   Name________________   Group______

(A) Voltage, Current, Impedance and Phase

Preliminary Data and Calculations

Measure r, the resistance of the inductor. Calculate the total resistance of circuit, R_{eq}. Calculate the theoretical values of the reactances and impedance of the system, given f = 120 Hz. Remember to show your work.

r (Ω)	R_{eq} (Ω)	X_L (Ω)	X_C (Ω)	Z (Ω)

Table A

	Experimental (measured)	Theoretical	percent Error	Phase Analysis	
$V_{S,max}$ (V)	blank	(measured)	blank	$t_{V,peak}$ (measured)	
I_{max} (A)				$t_{I,peak}$ (measured)	
$V_{R,max}$ (V)				$\Delta t = t_{I,peak} - t_{V,peak}$	
$V_{rL,max}$ (V)		*		Experimental ϕ (from Δt)	
$V_{C,max}$ (V)				Theoretical ϕ (from calculations)	
				percent Error	

* Remember, the inductor has resistance.

Recall: $percent\ error = \frac{|exp - theo|}{theo} \times 100\%$.

Show your work. Use the back of this sheet or a separate sheet if you need more space.

Qualitative Phase Analysis (circle answers)

Source voltage	• leads • lags	current by	• $\pi/2$ • less than $\pi/2$
Voltage across the resistor	• leads • lags • is in phase with	current	
Voltage across the inductor	• leads • lags	current by	• $\pi/2$ • less than $\pi/2$
Voltage across the capacitor	• leads • lags	current by	• $\pi/2$ • less than $\pi/2$

Question A.1: Do the peak values add up to the peak value of the source, that is, does $V_{S,\max} = V_{R,\max} + V_{rL,\max} + V_{C,\max}$? Should they add this way? Explain why or why not.

Question A.2: Look at the phase differences in the resistor, inductor, and capacitor. Are they what you expected? Discuss the rules and expectations of the phase difference.

between V_R and I,

between V_{rL} and I,

and between V_C and I:

Question A.3: Using your theoretical results, find the average power dissipated in this circuit. (Remember that you measured the peak values (amplitudes) and not the rms values.) Show work.

(B) Inductor with Ferromagnetic Core

I_{max} (A)	
$V_{L,\text{max}}$ (V)	

L (H) (with core)	

Using these values, compute the new value of the inductance. Show your work. (Hint: This is the reverse of the calculation of $V_{rL,\text{max}}$ in Table A.)

(C) Alternating Current, frequency, and Diodes

C.1. Describe what you see when the input frequency is very low (around 5 Hz).

C.2. Describe what you see as the input frequency increases.

C.3. Describe what you see when the frequency is high enough for the LED light to appear steady.

Question C.1. The LED housing does not contain a single LED. Review the results in part C.1 and C.2. What configuration of LEDs would explain these results?

Question C.2. Explain the results of part C.3 of the experiment. To do this, you may need to do a little research into the nature of human color vision.

Series RLC Circuit: Resonance

Equipment and Setup: RLC circuit board, Multimeter, 3 Banana leads, Capstone file – RLC Resonance.cap

Theory

In the previous experiment, we studied the behavior of a series RLC circuit. The table below reviews the impedance and phase for each of our three linear circuit elements when taken by themselves.

	Impedance - Z	Phase angle - ϕ
Just R	R	0
Just L	$X_L = \omega L = 2\pi f L$	$\frac{\pi}{2} = 90°$
Just C	$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$	$-\frac{\pi}{2} = -90°$

In a series RLC AC circuit we found the following expressions for the impedance and phase angle.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{and} \quad \tan\phi = \frac{X_L - X_C}{R}$$

At very low frequencies we can see that X_L is small and X_C is large and it follows that $X_L - X_C < 0$ and $\phi < 0$. For very high frequencies we can see that X_L is large and X_C is small and then $X_L - X_C > 0$ and $\phi > 0$. It should be clear that as the frequency is varied there will be some frequency where $X_L = X_C$ and $\phi = 0$; this is known as resonance. To find the *resonance frequency* f_0, we equate the reactances.

$$X_L = X_C \implies \omega_0 L = \frac{1}{\omega_0 C} \implies \omega_0 = \frac{1}{\sqrt{LC}} \implies f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

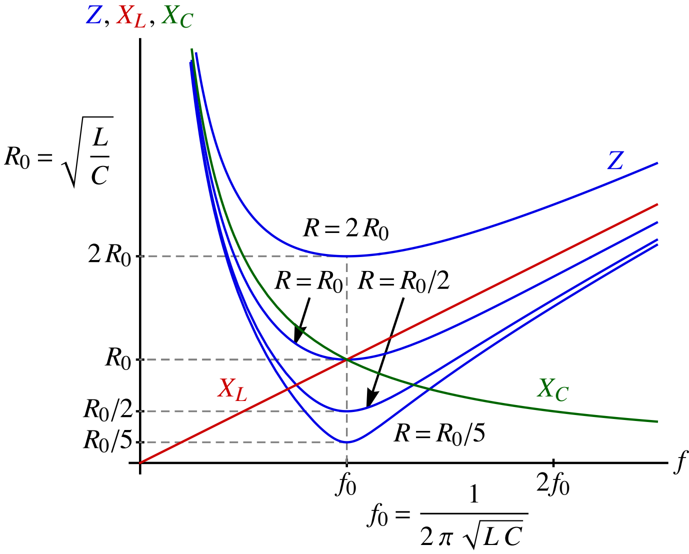
In addition to the phase angle vanishing $\phi = 0$, we also get a minimum impedance at resonance,

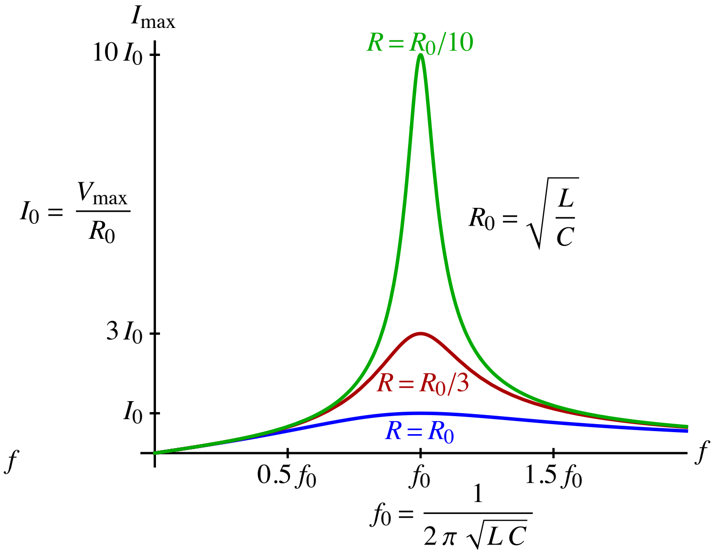
$$Z = Z_{\min} = R$$

since the reactances are equal $X_L = X_C$. In this experiment we will use these two signatures, the vanishing phase angle and minimum impedance, to find the resonance frequency. The relevant scale of resistance is the value of the reactances at resonance.

$$R_0 = \omega_0 L = \frac{1}{\omega_0 C} = \sqrt{L/C}$$

The smaller the resistance R, compared to R_0, the more dramatic the effects of resonance.





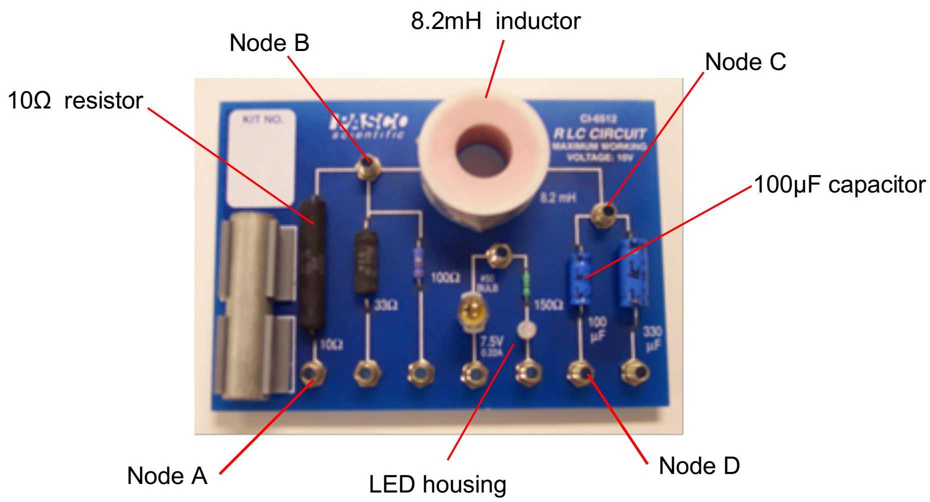
The graph on the left shows the frequency dependence of the inductive reactance (in red), the capacitive reactance (in green) and impedance (in blue). The impedance depends on resistance; curves for four different resistances are shown. The minimum impedance is the resistance, and that occurs at the resonance frequency.

Part A of the experiment will reproduce the current versus frequency graph on the right. We will fix the peak voltage V_{max} and vary the frequency. The peak current as a function of frequency is $I_{max}(f) = V_{max}/Z(f)$. The peak current will have its maximum value when the impedance is at its minimum and that is at resonance.

$$\max\, I_{max} = I_{max}(f_0) = \frac{V_{max}}{Z(f_0)} = \frac{V_{max}}{R}$$

We will reproduce this theoretical current versus frequency graph using the values of our circuit elements L, C and R. We will also take frequency-current data and plot those data points on the same graph.

Procedure



Preliminaries:

On the RLC circuit card identify the following components:

$R = 10\ \Omega$ $L = 8.2$ mH $C = 100\ \mu$F

Part A. Frequency-Current Data

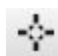
For a constant source voltage, you will plot I_{max} vs frequency. The resonance frequency can be identified by the location of the peak of this plot, the frequency at which I_{max} is maximum. You will be using the 8.2 mH inductor with the 100 μF capacitor. The only resistance in the circuit will be that of the coil, the inductor. This small resistance will provide a greater change in current, which will make the peak of the plot easier to discern.

(A.1) Use a multimeter with banana leads to measure the internal resistance r of the coil. This is the resistance between Node B and Node C. Record this value on the Data Sheet. Put the multimeter away; you won't need it again.

(A.2) Turn on the Pasco interface and open the Capstone file on your computer.

(A.3) Locate output 1 in the upper right corner of the Pasco interface. Using banana leads, connect the negative terminal of output 1 to Node B on the RLC circuit board and the positive terminal to Node D. The voltage source on the Pasco interface has built in voltage and current sensors. We will monitor the peak current I_{max} and make a plot of the peak current versus frequency.

(A.4) In the Capstone interface choose the Current Scope tab. Set the oscilloscope peak voltage to $V_{max} = 2.0$ V and keep it there.

(A.5) To find the peak current I_{max} use the data tool  in the Capstone interface as in the previous experiment. Fill in Table A.1 with frequency-current data, varying the frequency from 30 Hz to 300 Hz in increments of 30 Hz, as shown in the table. The peak current should reach its maximum at the resonance frequency. Identify where in your data set the current reaches its maximum.

(A.6) Around that maximum current from Table A.1, take data with frequency readings separated by 5 Hz. With this greater precision, identify the new maximum peak current. In Table A.2 record seven frequency-current data points with the frequencies separated by 5 Hz and with the middle value having the maximum current. (For instance if the maximum current occurs at 145 Hz, then record your readings every 5 Hz between 130 Hz and 160 Hz.)

(A.7) Proceed to the other parts of this lab. You will plot and analyze your data for part A afterwards.

Part B. Finding resonance using phase difference

Notice the dual-trace oscilloscope plot on your capstone display. The plot actually has two axes that are set exactly the same. The Pasco Interface source 1 actually has a built in voltmeter and current meter which will measure the voltage and current of the source. When the system is in resonance, the source voltage should be in phase with the current.

(B.1) Let's monitor the source Voltage and source Current at the same time. On the right side axis, left click the label "Output Voltage (V)" and change it to "Output Current (A)." Set the frequency on the signal generator to 120 Hz.

(B.2) Turn on the signal generator and click monitor. Adjust the frequency until the voltage and current are in phase (peaks and troughs occur at the same time). Record this frequency in part B of your Data Sheet. Stop monitoring and turn off the signal Generator.

Part C. Finding resonance using an x-y scope

An x-y oscilloscope reading displays one input vs another. This will display the current on the vertical axis vs the source voltage on the horizontal axis. The result is an elliptically-shaped figure called a lissajous (lee-*suh*-zhoo).

The plot is $V = V_{max}\cos(\omega t + \phi)$ vs. $I = I_{max}\cos(\omega t)$. At resonance, phase angle $\phi = 0$, and the lissajous collapses into the form of a slanted line.

(C.1) On the capstone display, select "x-y scope" tab (top, left). Set the frequency to 120Hz and turn on the signal generator.

(C.2) Click monitor and adjust the frequency of the Signal Generator until the lissajous on the x-y oscilloscope display collapses into a slanted line. Record the frequency on the data sheet.

(C.3) Add the iron core to the coil, changing its inductance. Repeat the previous step to find the new resonance frequency. You will find the unknown inductance from this.

(C.4) Stop monitoring and turn off the Signal generator.

When you put the equipment away, please put the circuit cards, banana leads, and voltage probes away **separately**. These do not go in the same bag!

Proceed to your calculations. Turn in your data sheet and calculations.

Resonance – Data Sheet   Name________________ Group______

Part A - Frequency-Current Data

L = 8.2 mH and C = 100 µF

Source Voltage V_{max} (V)	Coil Resistance r (Ω)
2.0	

Table A.1

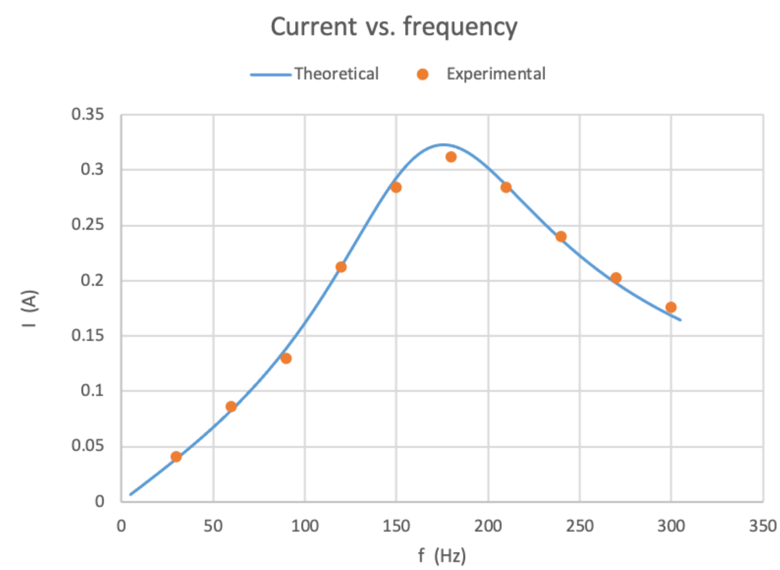
f (Hz)	I_{max} (A)
30	
60	
90	
120	
150	
180	
210	
240	
270	
300	

Table A.2

f (Hz)	I_{max} (A)

Data analysis for Part A.

In Excel (or some other equivalent spreadsheet) create a graph of current vs. frequency. Plot all the data listed in Table A.1 as data points but do not draw any curve with that data. On the same graph, plot the theoretical curve. To do this, use Excel to generate a list of frequencies varying from 5 Hz to 300 Hz in 5 Hz increments and then, using Excel formulas, find the corresponding theoretical values of the peak current. Calculating these theoretical values will involve using your measured value of the resistance r, as well as the listed values of the inductance and capacitance. Plot these theoretical frequency-current values, then right-click, choose Format Data Series and choose Smoothed line. (Note that to plot two data sets in the same graph, right-click Select Data to add a second data set.) The resulting graph should look like this. (The experimental values were taken from real data.)



For the data in Table A.2, plot current versus frequency, plot the best-fit parabola and show the equation of that parabola on the graph. (Because the data is near the graph's maximum, it will reasonably approximate a parabola.) You will likely need extra digits in the displayed equation for the trendline parabola. To do this right-click on the displayed equation in the chart and select "Format Trendline Label…" and choose enough decimal places to give each parameter at least three significant digits. With this graph is it fine to follow the Excel default and not put the origin (0,0) in the corner of the graph.

From the equation of the parabola for the graph of Table A.2, find the frequency where the current is the maximum, the resonance frequency f_0, and find the maximum peak current, max I_{max}. These are the experimental values for the Table A.3 below. Following the discussion in the theory section, find the theoretical values for f_0 and for max I_{max}.

Table A.3

Resonance frequency - f_0			Maximum peak current – max $I_{\max}$		
Experimental f_0 (Hz)	Theoretical f_0 (Hz)	Percent Difference	Experimental , max $I_{\max}$ (A)	Theoretical max $I_{\max}$ (A)	Percent Difference

Show your work. Use the back of this sheet or a separate sheet if you need more space.

Question A.1. Using your values, calculate the resistance scale factor $R_0 = \sqrt{L/C}$ discussed in the theory section and also calculate R/R_0 using the relevant resistance.

Question A.2. What causes the current to be maximum at resonance?

Question A.3. What if we had a constant current source (the current was always the same.) How would the amplitude of the voltage vary as the frequency is changed?

Part B - Finding resonance using phase difference

experimental (from Dual Trace scope) f_0 (Hz)	theoretical (from part A theoretical) f_0 (Hz)	percent error

Question B.1. What causes the phase angle to be zero at resonance?

Part C- Finding resonance using an x-y scope

Without Iron Core

experimental (from x-y scope) f_0 (Hz)	theoretical (from previous table) f_0 (Hz)	percent error

With Iron Core

frequency (with core) (from x-y scope) f_0 (Hz)	Inductance (with core) L (H)

Show your work.

Name ________________ Group ______

Geometric Optics and the Ray Box

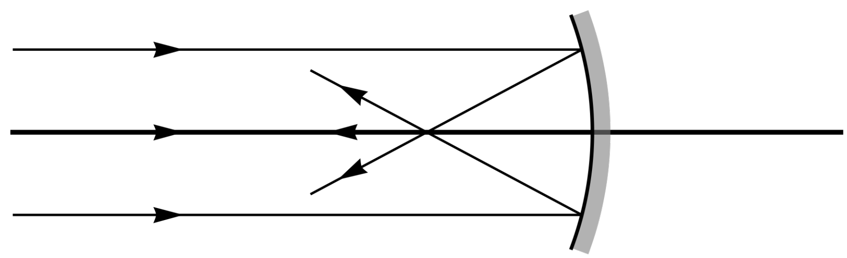
Equipment and Setup: Light source, Optical component set, Ruler, Protractor

The ray box can be used to get parallel rays of light from a source. For each part sketch the rays and components on a plain piece of paper. Sketch the components (mirrors and lenses) by tracing them with a pencil and draw the rays by marking two points on the ray with a pencil and then use a straight edge between the points to trace each line.

(A) Reflection from a Concave Mirror

Use three rays and a concave mirror. Align the central ray with the pre-drawn line and focus the rays to a point along the central axis. Trace the incident rays, the reflected rays and the mirror. Use the drawing to find the focal length of the mirror.

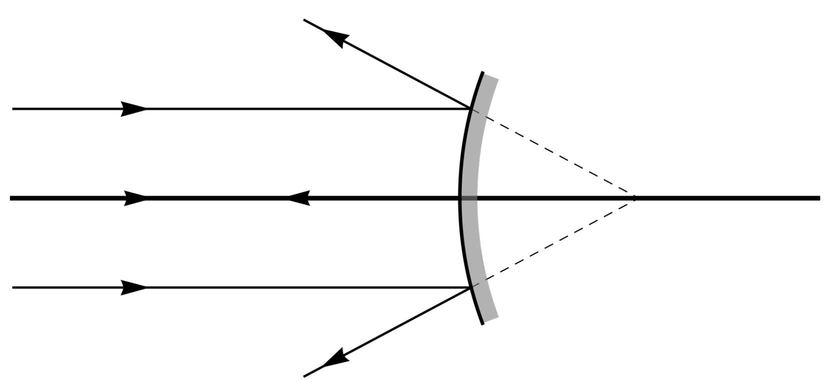
f = __________



(B) Reflection from a Convex Mirror

Use three rays and a convex mirror. Align the central ray with the pre-drawn line. Trace the incident rays, the reflected rays and the mirror. Extend the reflected rays backward to find where they intersect. Use the drawing to find the focal length of the mirror.

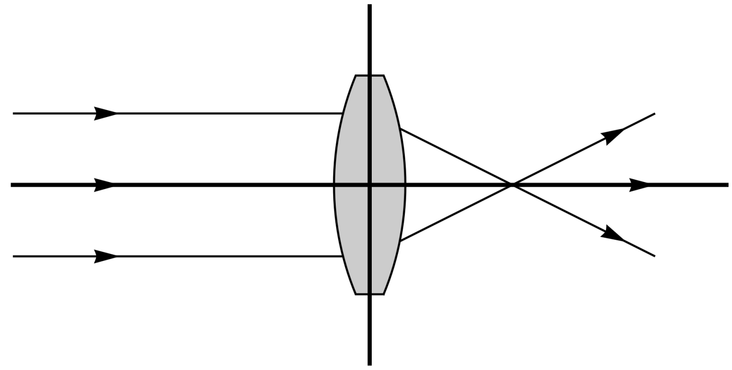
f = __________



(C) Double-convex (Converging) Lens

Use three rays and a double-convex lens. Align the central ray with the pre-drawn line and place the lens so that its center is along the pre-drawn perpendicular line. Focus the rays to a point along the central axis. Trace the incident rays, the refracted rays and the lens. Use the drawing to find the focal length of the lens. (Measure the distance from the center of the lens.)

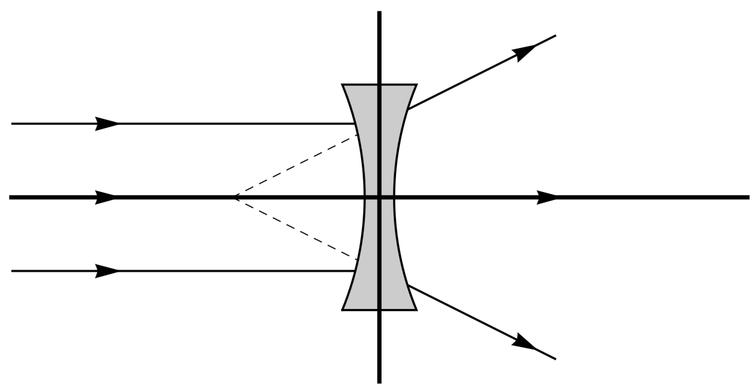
f = __________



(D) Double-concave (Diverging) Lens

Use three rays and a double-concave lens. Align the central ray with the pre-drawn line and place the lens so that its center is along the pre-drawn perpendicular line. The rays should diverge from the central axis. Trace the incident rays, the refracted rays and the lens. Extend the refracted rays backward to find the focal length of the lens. (Measure the distance from the center of the lens.)

f = __________



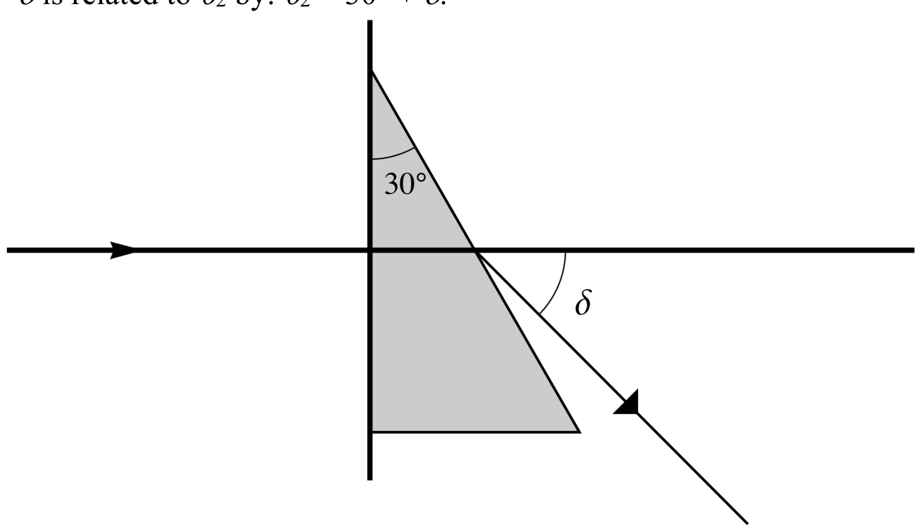
(E) Snell's Law

Use one ray and a prism with a 30° apex angle. Align the ray with the pre-drawn line and one face of the prism with the perpendicular. This makes the incident angle equal to the apex angle of the prism: $\theta_1 = 30°$. $n_1 = n$ is the unknown index of the prism and $n_2 = 1$ is the index of air. Draw the refracted ray (the ray leaving the prism) and measure the total angle of deflection δ. δ is related to θ_2 by: $\theta_2 = 30° + \delta$.

δ = __________ θ_2 = __________

Using Snell's Law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ calculate n, the index of the prism.

n = __________



(F) Plano--convex (Converging) Lens

Use three rays and the plastic lens with one flat face and one semicircular face. Align the flat face with the pre-drawn perpendicular line and the central ray with the long line. Trace the rays and the lens. From the diagram measure the focal length of the lens. (The distance is measured from the curved face.) Measure the radius of the lens.

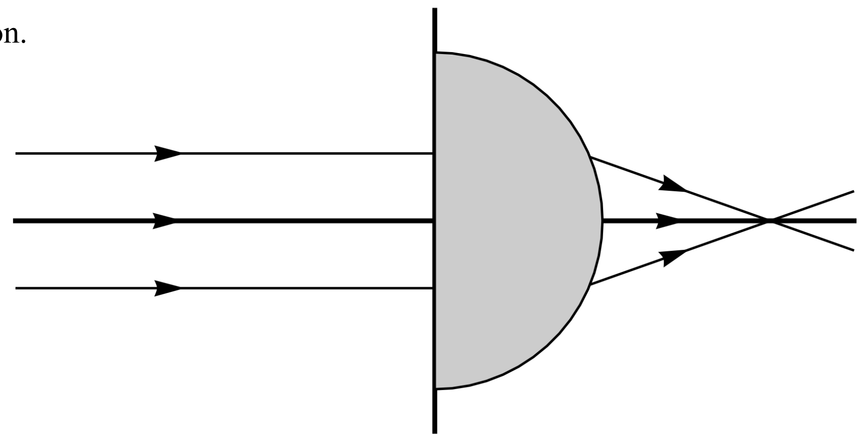
f = __________ R = __________

The lensmaker's equation $1/f = (n-1)(1/R_1 - 1/R_2)$ relates the radii of curvature of the faces of a thin lens and its index of refraction to the lens' focal length. In this expression the radii are measured from the side of the refracted ray. For this plano-convex lens we have: $R_1 = \infty$ (a flat face) and $R_2 = -R$. Plugging this into the lensmaker equation we get:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f} = (n-1)\left(0 + \frac{1}{R}\right) \Rightarrow n = 1 + \frac{R}{f}$$

Use this to find the lens' index of refraction.

n = __________



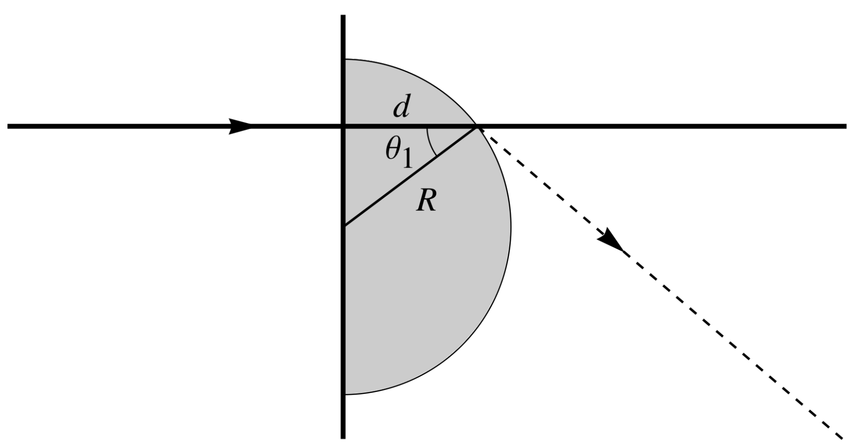
(G) Total Internal Reflection

Use one ray and the plano-convex lens from part (F). Align the flat face with the pre-drawn perpendicular line and the ray with the long line. Slide the flat face of the lens along the perpendicular until you reach the critical position where the refracted ray disappears. At that position carefully trace the curved face of the lens. Now the incident angle is the critical angle: $\theta_1 = \theta_{crit}$. Solve for this critical angle by measuring d (as shown) and by using $\cos\theta_1 = d/R$. Use this and $\sin\theta_{crit} = n_2/n_1 = 1/n$ to find the index of refraction of the lens.

d = __________

$\theta_1 = \theta_{crit}$ = __________

n = __________



Interference and Diffraction

Equipment and Setup: Optics Track (black), High Precision Diffraction Slits (single and multi-slit wheels), Red Diode Laser, Aperture Bracket, Linear Translator, High Sensitivity Light Sensor, Rotary Motion Sensor, *Capstone* program – Diffraction.cap

Theory

In this experiment we will shine laser light through a slide onto a screen. The images on the screen will look something like this:

The Image on the screen for single slit diffraction.

This image shows laser light hitting a wall after passing through a pair of narrow vertical slits.

Image on the screen for two slit interference

Single Slit Diffraction

When a monochromatic (single color) light of wavelength λ passes through a single narrow slit, it does *not* create a single bright streak of light (or a single bright spot for a laser). Diffraction, the light bending outward, coupled with interference creates a complex pattern of bright and dark fringes called a "Single Slit Diffraction Pattern." This looks like a series of bright stripes (for a solid light source such as a sodium lamp) or a series of spots (for a laser). Let the width of the slit be a, and the wavelength of the light be λ. The minima, the darkest regions of the pattern, are located at angles given by

$$a \sin\theta_m = m\lambda \qquad (m = \pm1, \pm2, \pm3 \ldots)$$

Where, θ_m refers to the angle of the mth diffraction minimum, that is, the m^{th} dark spot away from the principal maximum (bright central peak). Note that m cannot equal 0, since this is the location of the principal maximum. In addition, $+m$ and the $-m$ refer to orders on either side of the principal maximum. The position on the screen is given by

$$x_m = L \tan\theta_m$$

where $x = 0$ corresponds to the center of the pattern at $\theta = 0$. The length from the slit to the screen, L, is the distance from the center of the slit to the center of the pattern, at $x = 0$, on the screen.

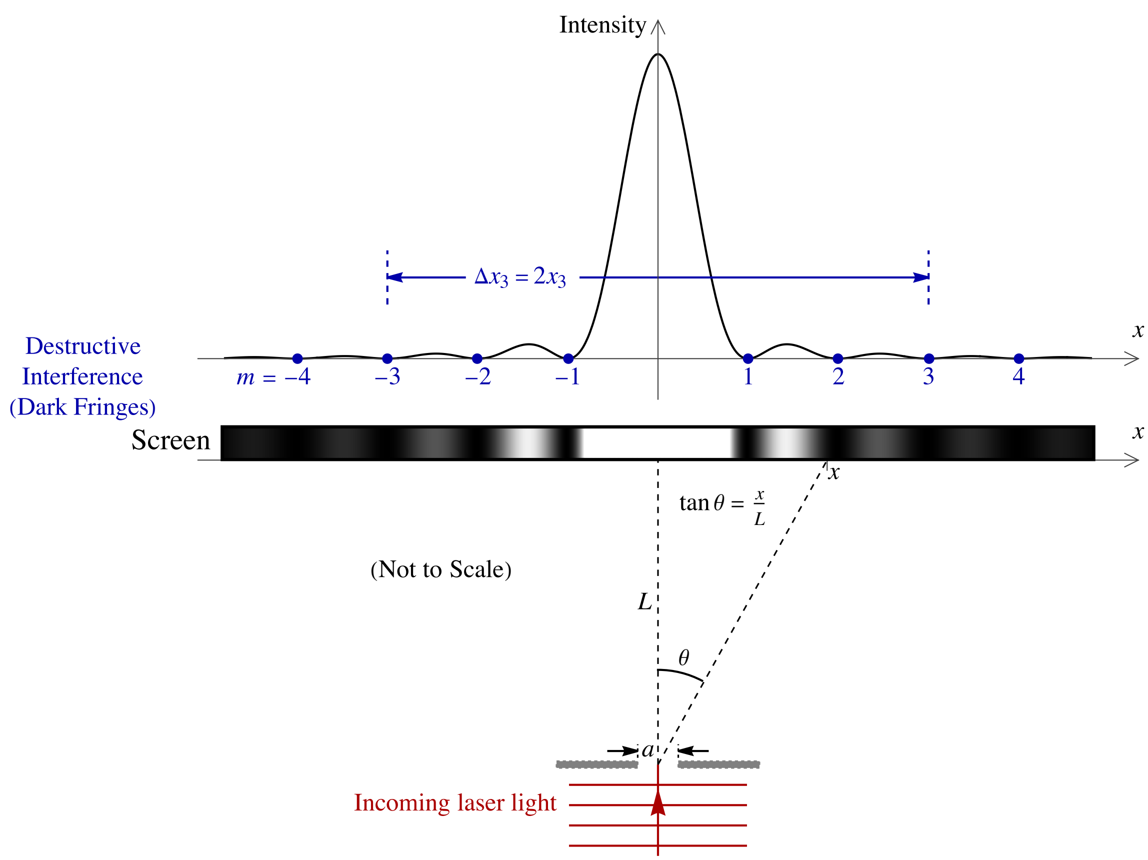
Since it may be difficult to determine the exact location of the center of the pattern, we will instead be measuring the distance between minima on either side of the principal maximum. For instance, in the diagram below, the distance Δx_3 is measured between the $m = -3$ and $m = +3$ minama. Since $\Delta x_m = 2x_m$ for any matching pair of minima, the angle between the center of the pattern and a given minima can be determined by

$$\tan\theta_m = \frac{\Delta x_m}{2L}$$

As it turns out, for our equipment, there is a fairly large uncertainty in the width of the slits. The uncertainty is ± 5 µm for any given slit, which corresponds to a 25% uncertainty for the 20 µm slit. As such, instead of using the slit width to estimate the wavelength of the laser, we will use the wavelength of the laser, which has a much smaller relative uncertainty, to estimate a more accurate value for the slit width.

(Note: generally the position on the screen is given by y and the distance between locations on the screen is given by Δy, even if the pattern spread is horizontal. We use Δx here in order to match the Delta reading on the *Coordinate Tool* in Capstone.)

The figure below is an illustration of the expected intensity of light versus position for a single slit light pattern. Below the intensity pattern is a drawing of the experimental setup; this shows the laser light passing through a slit of width a located a distance L from the screen and the light pattern on the screen. Note that the width a and distance L are not to scale. In the experiment, a is very small and L is very large. Important features are labeled.



The experimental setup and intensity pattern for a single slit of width a.

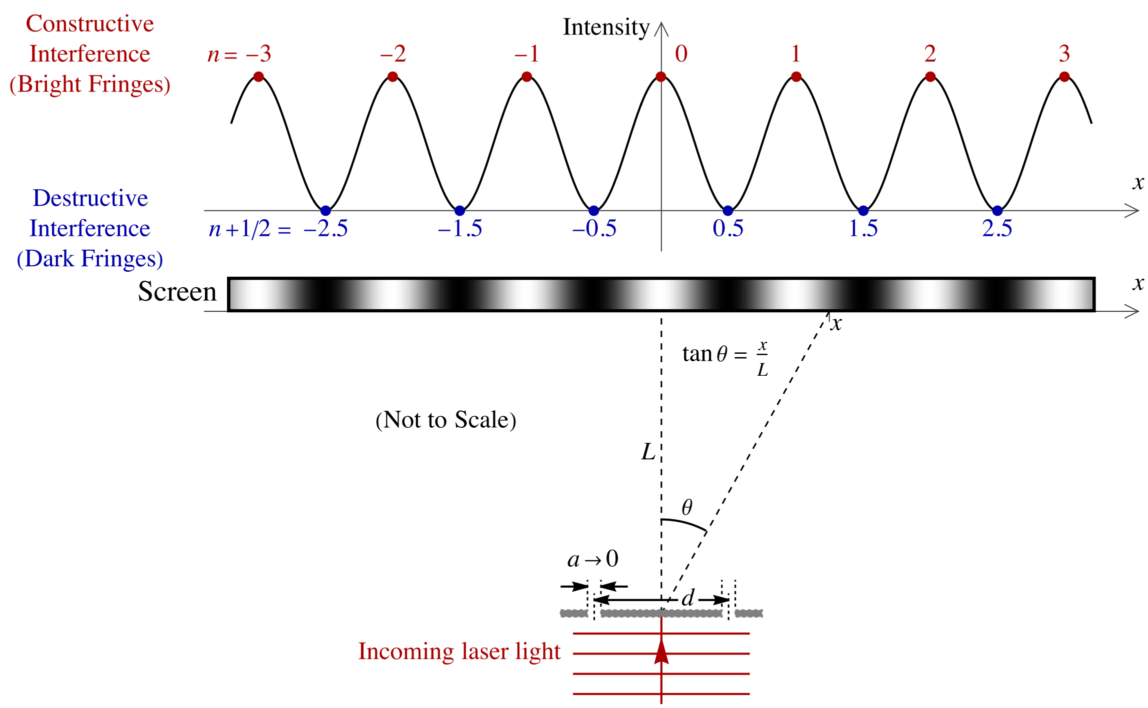
Double Slit Interference

When monochromatic light passes through a pair of narrow slits, the light from the two slits interferes to form a pattern of light and dark fringes (bright fringes appear as spots if the source is a laser). The slit spacing, d, is the distance between the centers of the two slits. The case of *pure* double slit refers to the idealization where the width of each slit a is infinitesimally small, $a \to 0$. The location of the bright fringes is given by

$$d \sin\theta_n = n\lambda \qquad (n = 0, \pm1, \pm2, \pm3 \ldots)$$

This looks similar to the formula given for single slit diffraction, except that this equation refers to the locations of the **maxima**, the centers of the *bright* spots! For clarity, we will use n to indicate the orders for the double slit interference pattern. In this experiment, we will concentrate on the double slit bright fringes and will not emphasize the dark fringes. The dark fringes occur when $d \sin\theta = (n + 1/2)\lambda$, where $n + 1/2 = \pm 1/2, \pm 3/2, \pm 5/2 \ldots$.

The intensity pattern for this pure double slit case is shown below.



The experimental setup and intensity pattern for a pure double slit with separation *d* and with negligible width.

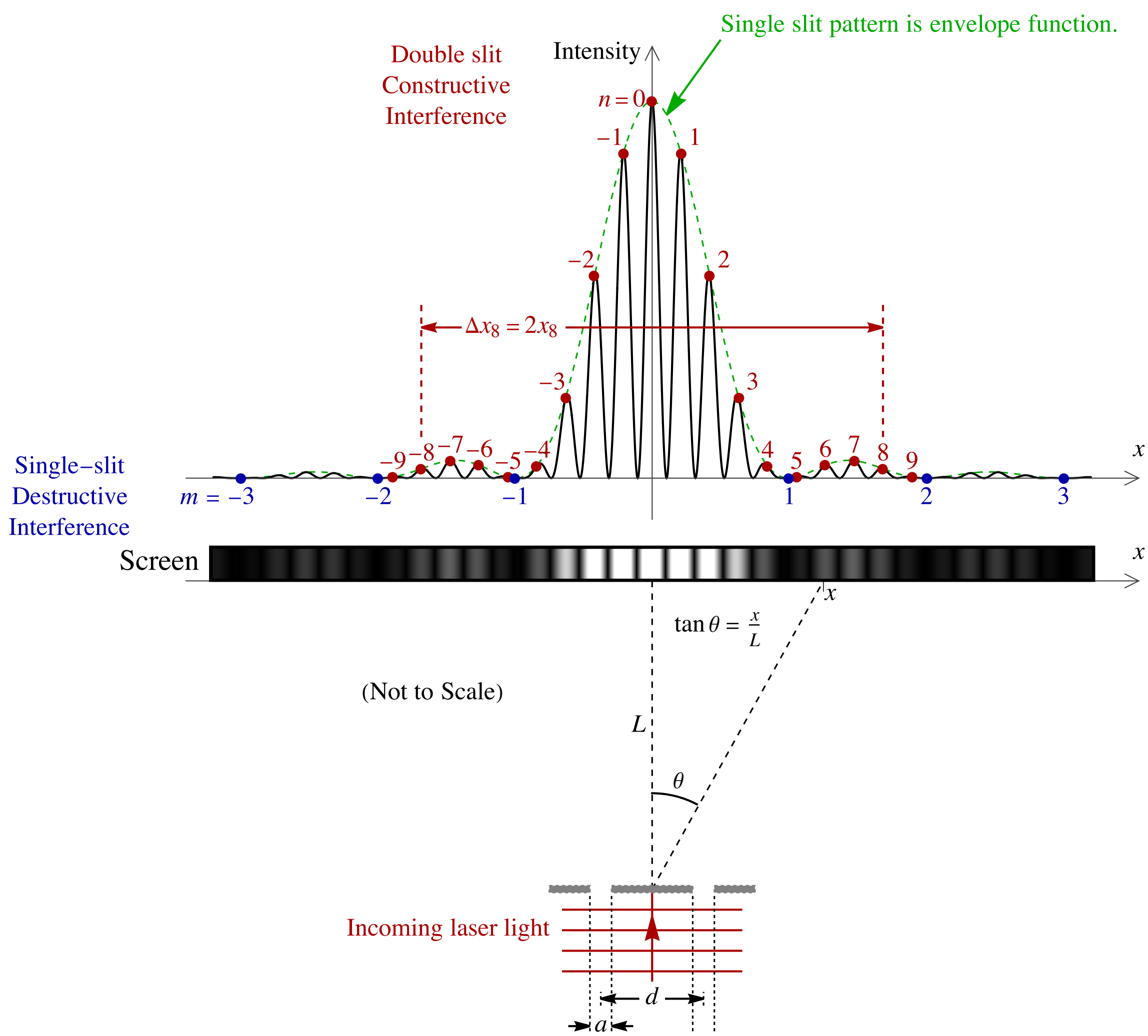
What we will study in this experiment is the case of double slit interference when the width of the slit a is not small compared with the slit spacing d. This gives rise to a much more complex pattern that turns out to be the pure double slit intensity pattern shown above sitting inside the single slit pattern; the single slit pattern is referred to as an *envelope function*.

Double slit interference patterns are more complex than single slit diffraction patterns because each slit has a width, a, and simultaneously produces a diffraction pattern. In the drawing below, the example graph of the interference pattern (solid line) shows that the intensity of the pure double slit pattern is modified by the single slit "envelope" (dashed line). This means that, for laser light passing through a double slit and shining on a distant screen, the pattern of bright spots will be very weak at large angels. In addition, some of the double slit maxima will be masked (missing) due to the single slit minima. When counting order number, n, it will be important to count these maxima as if they were actually visible.

The position on the screen corresponding to the n^{th} bright fringe (maximum) is given by $x_n = L \tan \theta_n$. Measurement of the angle θ_n can be found more accurately by measuring the distance between the $+n$th and $-n$th order maxima (given as $\Delta x_n = 2x_n$). The angle can be found by

$$\tan \theta_n = \frac{\Delta x_n}{2L}$$

Note that $x = 0$ is the location of the center of the principal maxima, $n = 0$, also known as the 0^{th} order bright fringe.

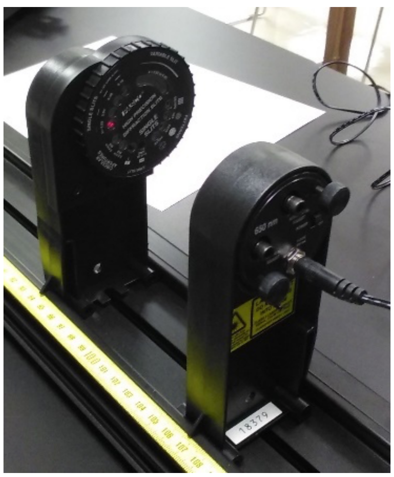


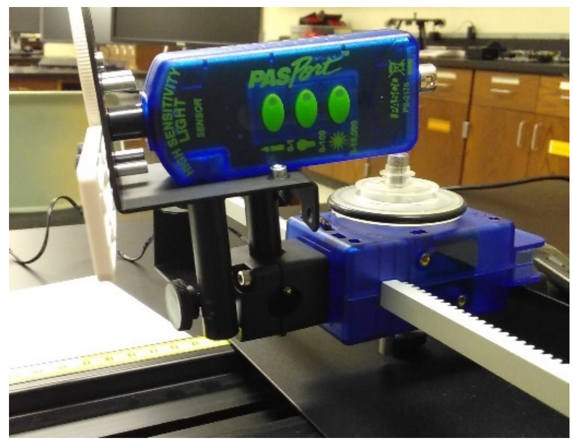
The experimental setup and intensity pattern for a double slit with separation *d* and with non-negligible width *a*.

Procedure

Setup

(S.1) Mount the laser on the end of the optics track by squeezing the clip at the base of the laser and snapping it into place. The laser should be at the far end from the computer, facing the computer. Mount the Single Slit Disk on the optics track in front of the laser with the printed side towards the laser. Turn on the laser. CAUTION: Never shine the laser beam directly into anyone's eye! Select the 0.02 mm aperture slit by rotating the disk until the desired slit is illuminated by the laser. Use the horizontal adjustment on the rear of the laser to center the laser on the slit. Turn off the laser while working on the next few steps.



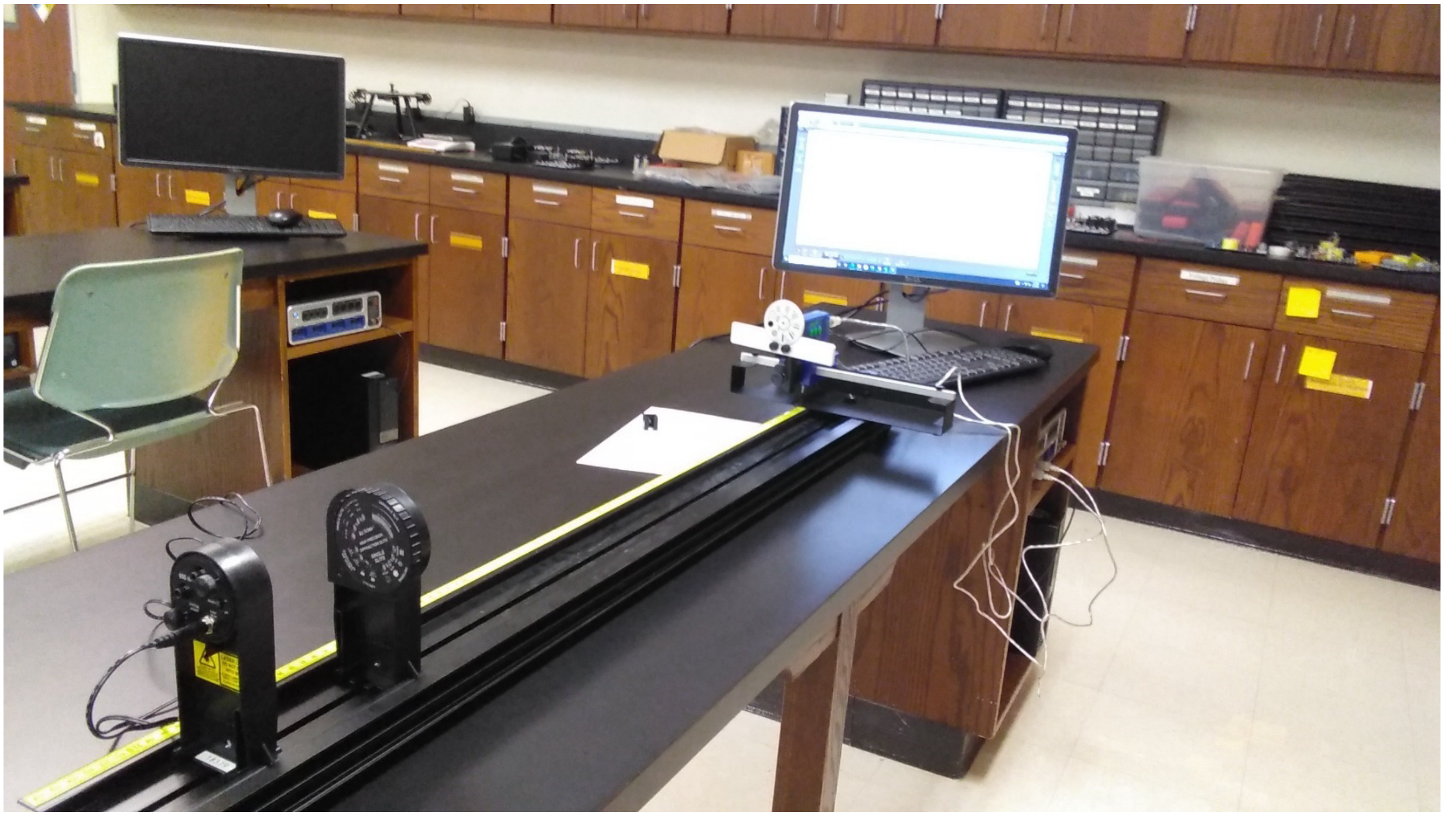


(S.2) Mount the Linear Translator on the optics track with the bracket facing away from the laser by sliding the square nut into the center slot of the track. Mount the Rotary Motion Sensor on the toothed rack of the Linear Translator. The toothed side should face away from the laser.

(S.4) Attach the Light Sensor to the Aperture Bracket using the 3 cm black plastic rod from the Light Sensor. There should now be two plastic rods attached to the base of the Aperture Bracket. Mount the Aperture/Light sensor assembly to the bracket on the Rotary Motion Sensor (image shown part-way inserted). If you insert the rod all the way, the Light Sensor will contact the rotary part of the Motion Sensor. To prevent this, pull up on the assembly a little, straighten the Aperture Bracket so that it is even (horizontal and facing the laser), then tighten the screw on the bracket to hold it in place. Set the Aperture Bracket to slit #6.

(S.5) Measure the length L, between the slits and the "screen" (the aperture bracket in front of the light sensor). Note that the tab on the side of the Slit disk is 2cm from the slits. You can use a straight edge against the aperture bracket to locate its position on the ruler. You can adjust the locations of the slit disk and the linear translator assembly to make the length L easier to read. Record this on your data sheet.

(S.6) Turn on the laser and use the vertical adjustment screw on the back of the laser to vertically center the diffraction pattern on the screen (that is, on the aperture bracket in front of the light sensor). Plug the Rotary Motion Sensor and the Light Sensor into PASPORT inputs 1 and 2 respectively. In the Physics – capstone folder, open the *capstone* file Diffraction.cap.

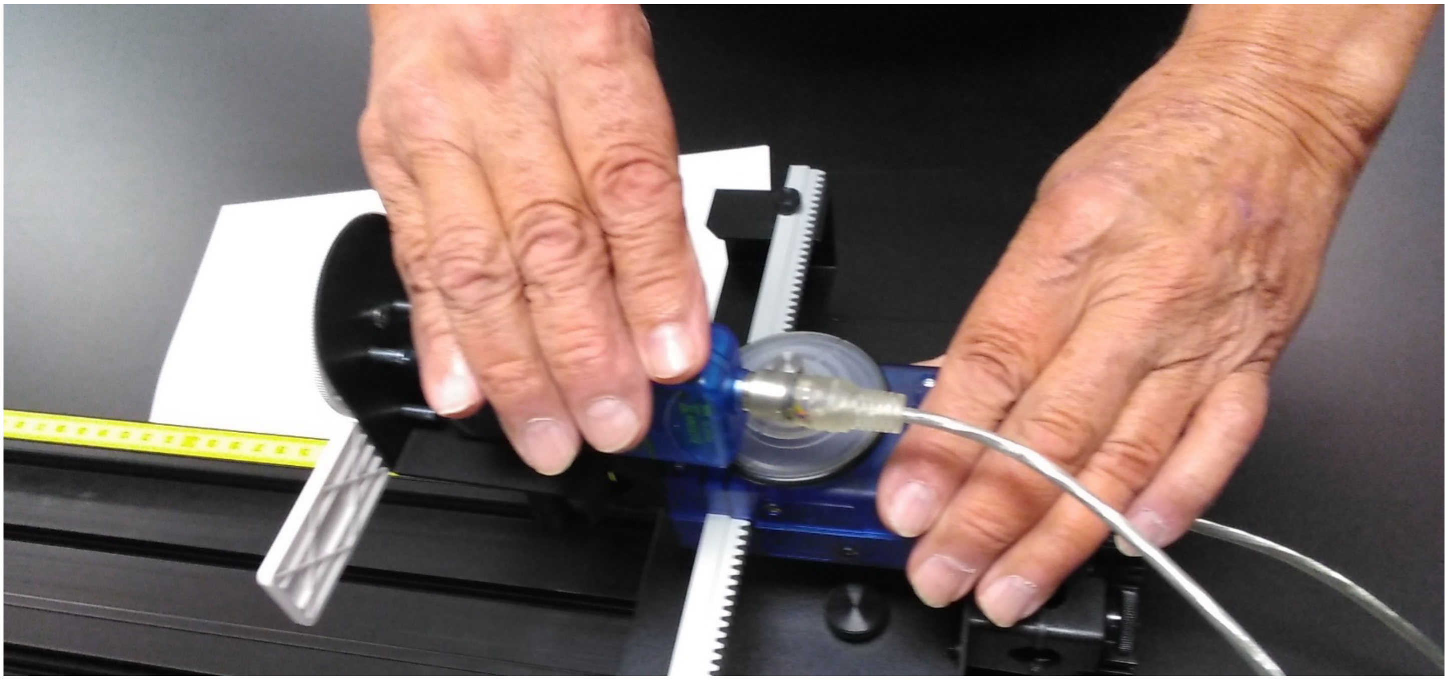


Part A. Single Slit Diffraction

(A.1) Read the wavelength, λ, on the back of the laser and record this on the data sheet. For best results, the lights in the room should be turned off. If this is not possible, you can proceed with the lights on. (The following is generally not necessary.) You can set the Light Sensor for maximum sensitivity by pressing the 0-1 button. If the light intensity is too high, you can turn it down by pressing the (0-100) button on the Light sensor.

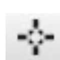
(A.2) Slide the light sensor assembly to the *start* position (looking from laser towards sensor, *start* is all the way to the left). Click *Record* (in capstone) and slowly move the light Sensor assembly all the way to the other side of the translator. Click *Stop* (in capstone) when you have finished the scan.

When sliding the Motion Sensor assembly, you can either push the assembly itself or you can slowly rotate the disk on top of the Motion Sensor. It helps to use your other hand to lightly hold down the light sensor to prevent vibrations. The slower you slide the assembly, the more data points you will get. You want to move slowly and evenly, without jitters. If the graph does not look smooth, delete the last run using the *Delete Last Run* button at the bottom of the screen and repeat the process. If the intensity maxes out (100%), change the gain setting on the light sensor and repeat the run. Click on *Data Summary* on the left of the screen, double-click your run number and re-label it “0.02 mm.” Click *Data Summary* to close it.



(A.3) To expand the graph, click the *Scale-to-Fit* button bon the far left of the graph toolbar (above the graph – you may need to click on the graph first to see it). There may be some noise near the center of the graph. This is a result of poor collimation of the solid state laser. You can ignore it as it does not affect the locations of the minima. You may also notice that the right and left sides of the graphs do not look even. The base of one side may be slightly raised due to background lighting. If possible, turn off the lights in the room. Otherwise just bear with it.

(A.4) You will need to expand the vertical scale of the graph to identify the minimum intensities. To do this, hover your mouse pointer on the vertical axis so that the pointer becomes a double arrow. Click and drag to expand the scale until you can clearly detect the locations of minimum intensity on either side of the central maximum.

(A.5) Click the *Coordinate Tool*  from the graph toolbar. Drag the crosshairs to the $m = -1$ minimum on the left side of the central maximum. Right-click the *Coordinate Tool* and select *Show Delta*. Drag the delta to the $m = +1$ minimum on the right side of the central maximum. Note: If the Δx value does not have 4 significant figures, right-click on the *Coordinate Tool*, go to properties, and change the number of significant figures to 4. Record Δx in the first row of table for Part A. Compute the width of the slit, a and record it in the table. Warning: Take note of the different units of length used throughout the lab. You can make the calculation easier by converting everything to mm (including λ and L) prior to calculation.

(A.6) The accuracy of the data increases by using minima further from the center. Select the farthest two minima you can see and drag the delta tool to bracket these. Remember, you need to use the same order on each side, so if you can see the 2nd order on one side and the 3rd order on the other, use the 2nd order minima. Record m and Δx in table for Part A and compute slit width, a.

(A.7) Rotate the Single Slit disk to select the 0.04 mm wide slit. Slide the Sensor assembly back to the starting position and record data. Label the run "0.04 mm." Using the *Coordinate Tool*, record Δx for three different pairs of minima. Record your results and compute slit width a.

Part B. Double Slit Interference

(B.1) Take note of the location of the Single Slit disk on the ruler of the optics track. Remove the holder by gently squeezing the clip at the base and place the Multi Slit disk in the same location. The slits should be facing the laser, so that the distance, L, to the screen remains the same. Set the disk to $a = 0.04$ mm and $d = 0.25$ mm setting.

(B.2) Record data for this setting. The pattern is more detailed, so you will need to move the detector very slowly and smoothly to get good data. Label this run "0.04a-0.25d."

(B.3) First you will verify the slit width, a. Expand the data until you can clearly detect the locations of the diffraction minima. Select the farthest pair of single slit minima you can clearly discern. Using the *Coordinate Tool*, record Δx. Record your results in table for Part A and compute slit width a.

(B.3) Find slit spacing, d: If needed, readjust the horizontal and vertical scales so that you can clearly see the locations of the interference *maxima*. For three different sets of order number, n, use the Coordinate Tool to measure Δx.

Remember:

- You are measuring between MAXIMUM intensity points (peaks). For the purposes of this lab, you can assume that the peaks occur in the exact middle between consecutive interference minima.
- Larger order numbers, n, will give more accurate results.
- Some interference maxima will be suppressed by the single-slit diffraction envelope. When counting order number, n, you need to count these maxima as if they were visible.
- Δx should have 4 significant figures. If you need to adjust this, see the instructions in step (A.5) above.

(B.4) Record your values of n and Δx in table for Part B. Compute values for the slit spacing, d.

Int./Diff. – Worksheet Name____________ Group______

Wavelength of laser (back of laser) λ __________________ Distance to "screen" L ______________
(don't forget units)

Part A – Single Slit Diffraction – computing slit width

Run	Labeled a (mm)	d (mm)	m	Δx (mm)	Calculated a (mm)
0.02 mm	0.02	N/A			
0.02 mm	0.02	N/A			

average a_{ave} =____________ mm percent difference between a_{ave} and 0.02 mm _____________________

Run	Labeled a (mm)	d (mm)	m	Δx (mm)	Calculated a (mm)
0.04 mm	0.04	N/A			
0.04 mm	0.04	N/A			
0.04 mm	0.04	N/A			

average of 1st three runs a_{ave} =____________ mm

percent difference between a_{ave} and 0.04 mm _____________________

Part B – Double Slit Interference – computing slit spacing

Run	Labeled a (mm)	d (mm)	m	Δx (mm)	Calculated a (mm)
0.04a-0.25d	0.04	0.25			

percent difference between a and 0.04 mm __________________

Run	Labeled a (mm)	d (mm)	n	Δx (mm)	Calculated d (mm)
0.04a-0.25d	0.04	0.25			
0.04a-0.25d	0.04	0.25			
0.04a-0.25d	0.04	0.25			

average d_{ave} =____________ mm percent difference between d_{ave} and 0.25 mm _____________________

Question 1 How did your values of a and d compare to the expected values. Were your percent differences low? If not, can you explain why? Were you able to verify the slit width, a of the double slit (Part B, 1st table)?

Question 2 When computing a and d, what units must L, λ, and Δx be in? Can they be in different units? Must they all be in meters? Explain (that is, don't just answer yes or no, be clear about why you believe this is so.)

Question 3 What are the units of m and n?

Question 4 Must θ be in radians, in degrees, or can it be in either radians or degrees. Explain, or give a mathematical example which illustrates your reasoning.

Problem 1 A physics instructor wants to produce a double-slit pattern large enough for her class to see. For the size of the room, she decides that the distance between the successive bright fringes should be at least 1.5 cm. If the slits have a separation distance of 35 μm, what is the minimum distance from the slits to the screen when 630 nm light is used?
Note: for this problem you can assume that the instructor is measuring near the center of the pattern, where the angle is small. When the angle is sufficiently small, the interference maxima are evenly spaced and we can use the small angle approximation that: $\sin\theta \approx \tan\theta = \frac{x_m}{L}$.

Problem 2 In a particular double slit interference pattern, the second bright fringe to one side of the central maximum occurs at an angle of 1.32°. What is the ratio of the second slit separation, d, to the wavelength of the light? (This is a fraction, answer in decimal form. Do not write this as a ratio!)

Problem 3 A screen is placed 1.2 m behind a single slit. In the resulting diffraction pattern, the distance between the two third-order minima is 3.6 cm. What is the distance between the two fifth-order minima?

Problem 4 What is the maximum number of single slit diffraction minima that will be produced on either side of the central maximum if green light with wavelength $\lambda = 553$ nm in incident on a single slit of width 8 μm? (Hint: the angle must be less than 90° so that $\sin\theta < 1$.)

Appendix – Resistor Color Code

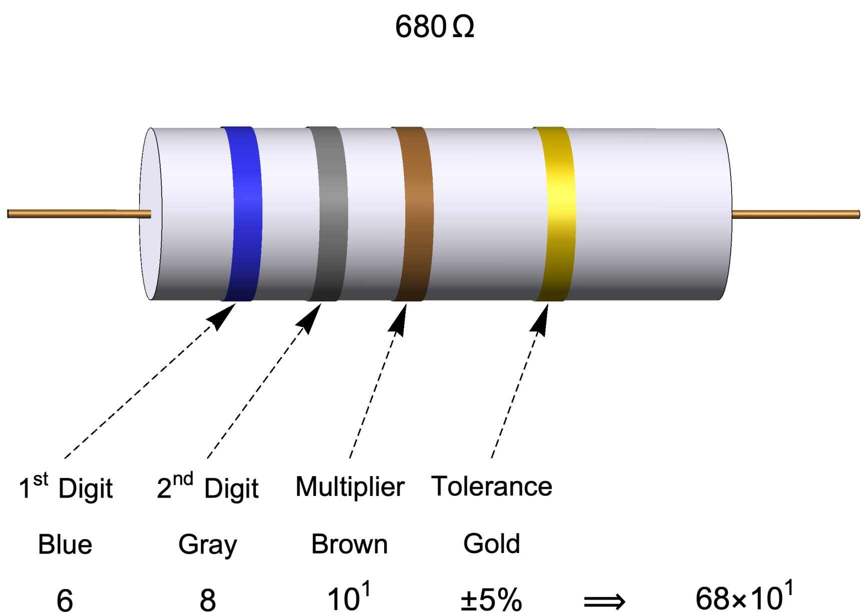
Resistors have a series of four, sometimes five, stripes which can be read to give the value of the resistance and also the tolerance, or relative uncertainty, which is given as a percentage.

To read a resistor with four stripes, first align the resistor so that the tolerance, the fourth stripe, as at the right. It should be separated from the other stripes by a larger distance, but this is not always sufficiently clear. If there is a *gold* or *silver* stripe, then that is the tolerance and goes to the right.

The first two stripes give digits, as shown in the table below. Suppose, as in the example, the first two stripes are *blue* and *gray*. From the table we read off the digits 6 and 8. The third stripe is the multiplier; this uses the same color to digit convention but now this digit is the power of ten. In the example, the third stripe is *brown* and since *brown* corresponds to the digit 1 the multiplier is 10^1. We can then read the resistance as 68×10^1 or 680 Ω. The fourth stripe is *gold* and that says that the resistance value is accurate to ±5%.

Color	Digit	Multiplier
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Color	Tolerance
Brown	±1%
Red	±2%
Gold	±5%
Silver	±10%



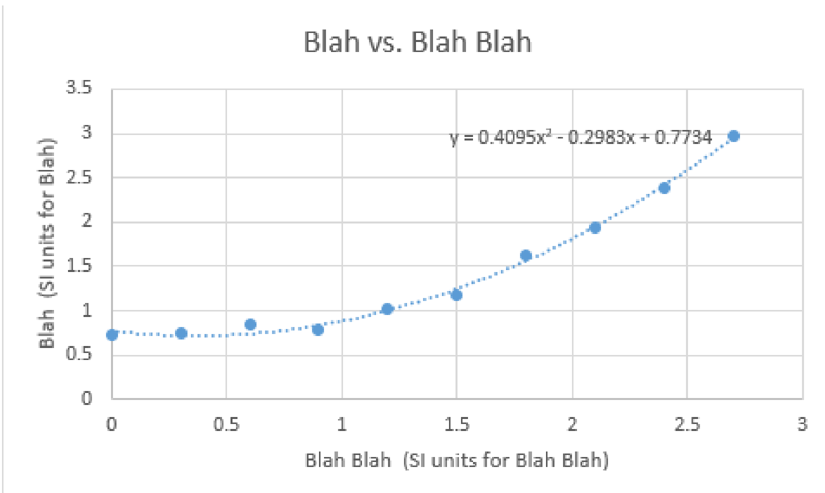
For a five-stripe resistor, it uses the same convention but has a third digit. As an example, suppose the colors are: *red*, *black*, *yellow*, *orange* and *red*. This reads as: 204×10^3 or 204 kΩ and the final *red* stripe implies that this value is accurate to ±2%.

There is an interactive version of this diagram in the Mathematica subfolder of the Physics folder on your laboratory desktop. The file name is ResistorColorCode.nb .

Appendix - Notes on Graphing

General Comments

- All graphs must be created on a computer using Excel or some other graphing software.
- The graph must be properly labeled.
  - At the top of each graph there should be a title describing the graph; this should include the variables that are plotted, for example a distance versus time graph could be labeled as "distance vs. time" or just as "x vs. t". It should also distinguish the graph from the other graphs in your experiment's write-up; for example, if there are multiple x vs. t plots then the title should read something like: "x vs. t for Part (A)"
  - Each axis must be labeled, and the units should be included in brackets. For the example of a distance x use the label: "x (m)".
- When plotting y vs. x, the horizontal axis is x. Always think y vs. x.
- All graphs must have a uniform scale along both axes. The axes should cross at the point (0,0).
- For any graph you must plot the **data points**, the **best-fit curve** and the **equation of the best-fit curve**.



Graphing with Excel

- When using Excel always use the scatter plot format. Other formats will likely cause problems with maintaining a uniform scale.
- Put the data in columns; if it is a graph of y vs. x, then the x column is on the left and y column is on the right. Highlight the data and then click the **Insert** tab. Then **Chart** > **Scatter** then choose the option "Scatter with only markers".
- There is a **Chart Design** menu item at the top that will return you to the chart ribbon.
- Select the **Add Chart Element** tab (at the top left of the ribbon) to add the labels. To label Axes click **Axes Titles**. There you will find **Primary Horizontal** and **Primary Vertical**, click on one then then the next and appropriately label each axis. Labeling the graph is done with the **Chart Title** choice or you can just click on the placeholder title and edit that.
- If the axis origin is not through (0,0) you can force it. Under the **Add Chart Element** tab click **Axes**, and then **More Axis Options**. This opens a **Format Axis** toolbar at the right. When there under Vertical axis crosses, uncheck **Automatic** and check **Axis value** and add (0,0).

- If you want to plot two data sets on the same graph right-click **Select Data** and then click to add a new set.
- Now you must add the best-fit line or trendline. Once again, go to the **Add Chart Element** tab and click **Trendline** > **More Trendline Options**. That opens a **Format Trendline** toolbar at the right. Under **Trendline Options** choose **Linear** for the best-fit line or choose **Polynomial** then **Order** 2 for the best-fit parabola. At the bottom of that toolbar always check **Display Equation on chart**. Sometimes the equation given for the trendline lacks significant figures. For instance, suppose one of the parameters in the trendline is 0.000003, although there are seven digits displayed, there is only one significant figure. To get more digits displayed then right-click on the trendline and choose **Format Trendline Label**, then choose an appropriate number of decimal places to get enough significant digits.
- Remember always that a graph of something versus something else, the first thing is the y axis and the second is the x. It is always y vs. x.
- If you mistakenly reverse the data when plotting you can swap the x and y values instead of starting the graph from scratch. To the right on the **Chart Design** ribbon choose **Select Data**. There you can manually change the references for the x and y values.