

Chapter J

Electromagnetic Radiation

Blinn College - Physics 2326 - Terry Honan

In this chapter we discuss electromagnetic radiation. This includes: radio waves, microwave, infrared radiation, visible light, ultraviolet radiation, x-rays and gamma rays. As an immediate consequence of Maxwell's equations in a vacuum we get a wave equation. We will derive this using the integral form of Maxwell's equations; this is awkward but the result is important. As an aside, we will discuss the derivation using the differential form of Maxwell's equations. From the wave equation we will solve for the speed of these electromagnetic waves in terms of our electromagnetic constants.

$$v = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

After deriving the wave equation, we will discuss the energy and momentum carried by this radiation and also its intensity and pressure. This chapter serves as a bridge between electromagnetism and optics.

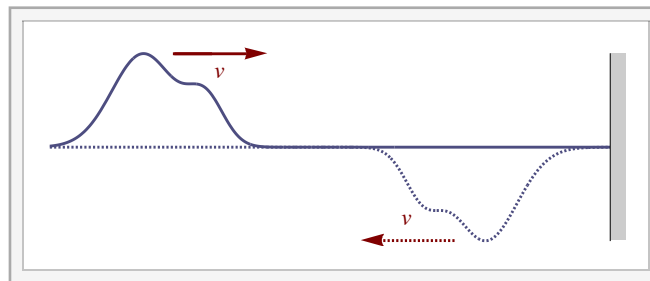
J.1 - The One Dimensional Wave Equation

We will take some generic wave variable to be u which is some disturbance that varies as a function of x , the position, and t , time. For example, consider waves on a stretched string. The position along the string is labelled by x , t is time and u is the distance of a point on the string from its equilibrium position. The one dimensional wave equation is

$$\frac{\partial^2}{\partial x^2} u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u,$$

where v is the wave speed. This is a second order partial differential equation for $u(x, t)$.

The General Solution

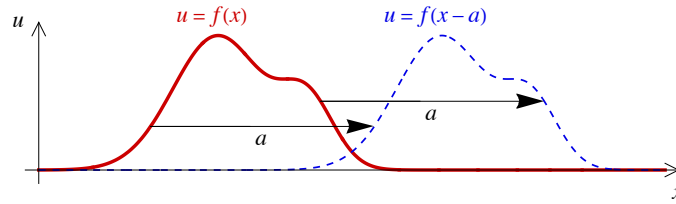


Interactive Figure

The general solution of an ordinary differential equation (ODE), where we solve for functions of one variable, involves arbitrary constants. The general solution of a partial differential equation (PDE), where we are solving for functions of several variables, will involve arbitrary functions. The general solution is

$$u(x, t) = f(x - vt) + g(x + vt),$$

where f and g are arbitrary functions. To understand this general solution, consider the function $u(x) = f(x)$. If we shift this by a in the positive direction we get $u(x) = f(x - a)$. We can now see that $u(x, t) = f(x - vt)$ describes a pulse of arbitrary shape $u(x) = f(x)$ moving in the positive direction with speed v . $u(x, t) = g(x + vt)$ corresponds to a pulse of a different arbitrary shape moving in the opposite direction at the same speed.



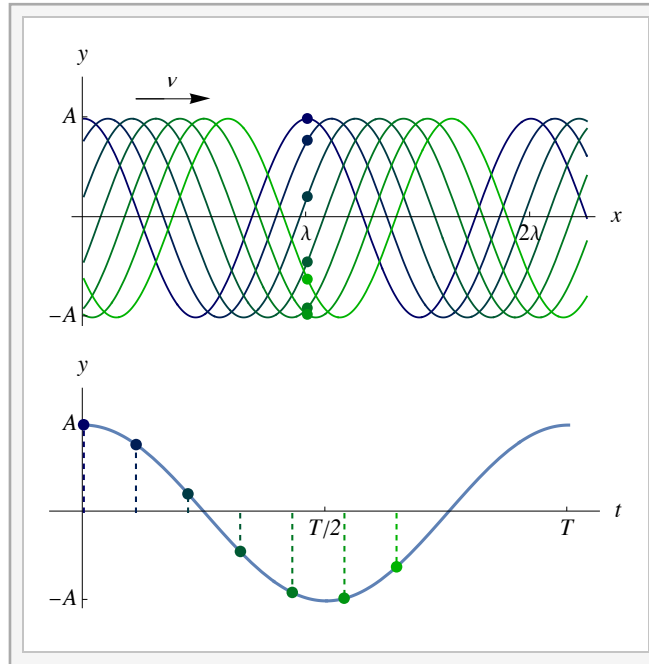
To verify this is a solution we will plug $u(x, t) = f(x - vt)$ into the wave equation. We need to evaluate the partial derivatives using the chain rule:

$$\frac{\partial^2}{\partial x^2} f(x - vt) = \frac{\partial}{\partial x} f'(x - vt) = f''(x - vt) \text{ and}$$

$$\frac{\partial^2}{\partial t^2} f(x - vt) = (-v) \frac{\partial}{\partial t} f'(x - vt) = (-v)^2 f''(x - vt).$$

Inserting $u = f(x - vt)$ into the wave equation we can see now that it is a solution for any function f . If we replace v with $-v$ it is still a solution and since f is arbitrary we can replace it with g ; it follows that $u = g(x + vt)$ is also a solution. Since the derivative of the sum of two functions is the sum of the derivatives the sum of our f and g solutions must also be a solution. This verifies that our expression for the general solution is indeed a solution. For it to be the general solution then *any* solution can be written in this form; to verify this is beyond the scope of the class.

Sinusoidal Waves



We often consider waves where the shape of the pulse f (or g) is sinusoidal.

$$f(x) = A \cos kx$$

A is called the amplitude. k is called the wave number; this is related to the wavelength λ , which is the spatial period of the function. Since the period of cosine is 2π and the period of f is λ we get $k\lambda = 2\pi$ or

$$k = \frac{2\pi}{\lambda}.$$

If we take this function f and move it in the positive or negative direction we get $f(x \mp vt) = A \cos[k(x \mp vt)]$ or

$$u(x, t) = A \cos(kx \mp \omega t),$$

where the angular frequency ω and wave number are related to the wave speed by $kv = \omega$. Since the angular frequency is related to the frequency by $\omega = 2\pi f$ the wave speed can also be written in terms of the frequency and wavelength.

$$v = \frac{\omega}{k} = f \lambda$$

A less mathematical derivation is to observe that the wave moves one wavelength in one period, $v = \lambda/T$, and then use the fact that the period and frequency are related by $f = 1/T$.

Example J.1 - Sinusoidal Waves

A wave of the form

$$u(x, t) = (0.020 \text{ m}) \sin[(105 \text{ s}^{-1})t + (3.0 \text{ m}^{-1})x]$$

travels down a string. What are the frequency and wavelength of the wave? Also, what is the wave speed and what is the direction of the wave?

Solution

From the form of the function we can read off the amplitude A , the angular frequency ω and wave number k .

$$A = 0.020 \text{ m}, \quad \omega = 105 \text{ s}^{-1} \quad \text{and} \quad k = 3.0 \text{ m}^{-1}$$

The frequency, wavelength and speed follow from formulas for sinusoidal waves.

$$f = \frac{\omega}{2\pi} = 16.7 \text{ Hz}, \quad \lambda = \frac{2\pi}{k} = 2.09 \text{ m} \quad \text{and} \quad v = \frac{\omega}{k} = 35 \frac{\text{m}}{\text{s}} = f \lambda$$

The solution for a pulse is $f(x \mp vt)$, where the negative sign means the pulse is moving in the positive- x direction and positive implies the negative- x direction. Since the relative sign between ωt and kx terms is positive, the wave is moving in the negative- x direction.

J.2 - Plane Waves in a Vacuum

Maxwell's Equations in a Vacuum

In a vacuum there is no matter so it follows that there is no charge or currents.

$$Q = 0 \quad \text{and} \quad I = 0$$

Removing the source terms from Maxwell's equations we get

$$\begin{aligned} \oint \vec{E} \cdot d\vec{A} &= 0 \\ \oint \vec{B} \cdot d\vec{A} &= 0 \\ \oint \vec{B} \cdot d\vec{r} &= \mu_0 \varepsilon_0 \frac{d}{dt} \Phi_e \\ \oint \vec{E} \cdot d\vec{r} &= -\frac{d}{dt} \Phi_m \end{aligned}$$

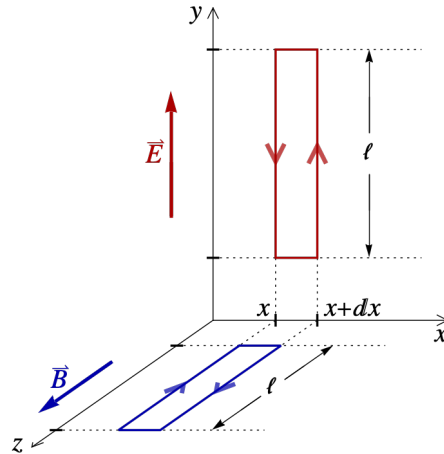
Plane Wave Assumption

A plane wave is a one dimensional wave in three dimensional space. If the wave propagates in the x -direction then we assume that nothing depends on the variables y and z , making the fields constant on a plane. This makes the problem one dimensional.

We will make an additional assumption, which will be justified when the solution is verified, that the electric and magnetic fields are mutually perpendicular and both are perpendicular to the direction of propagation. Take the electric field to be in the y -direction, the magnetic field to be in the z -direction and the x -direction to be the direction of wave propagation. The fields then take the form:

$$\begin{aligned} \vec{E} &= \hat{y} E(x, t) \\ \vec{B} &= \hat{z} B(x, t). \end{aligned}$$

Maxwell's Equations with the Plane Wave Assumption



Applying the plane wave assumption to Maxwell's equations in a vacuum will give us a pair of coupled first order equations. Combining these equations will give a one dimensional wave equation.

First apply Faraday's law to a contour in the xy -plane. Take the contour to be of length dx in the x -direction and length l in the y -direction. Now evaluate the line integral of the electric field around this contour. The segments in the x and $-x$ directions contribute 0 since the segment is perpendicular to the field. The segment in the positive y -direction at $x + dx$ contributes a $E(x + dx, t) l$ term and the segment in the negative y -direction at x gives a $-E(x, t) l$ term.

$$\oint \vec{E} \cdot d\vec{r} = E(x + dx, t) l - E(x, t) l = \frac{\partial E}{\partial x} l dx$$

Here we used the definition of the derivative, which can be written as:

$$f(x + dx) - f(x) = \frac{df}{dx} dx.$$

The right hand side of Faraday's law involves the derivative of the magnetic flux. The magnetic field is in the z -direction which is the same as the direction of the positive normal to the surface. The magnetic flux becomes $\Phi_m = B l dx$ and the time derivative becomes a partial time derivative. Faraday's law becomes:

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \Phi_m \implies \frac{\partial E}{\partial x} l dx = -\frac{\partial B}{\partial t} l dx \implies \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}.$$

We may similarly apply Ampere's law in the xz -plane to a contour of length dx in the x -direction and length l in the z -direction.

$$\oint \vec{B} \cdot d\vec{r} = B(x + dx, t) l - B(x, t) l = \frac{\partial B}{\partial x} l dx$$

The electric flux becomes $\Phi_e = -E l dx$ where we have the negative sign because the positive normal is in the negative y -direction which is opposite the direction of the electric field. Ampere's law then gives:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \epsilon_0 \frac{d}{dt} \Phi_e \implies \frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}.$$

The Wave Equation and the Speed of Light

Maxwell's equations with the plane wave assumption gave us a pair of coupled first order partial differential equations.

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \quad \text{and} \quad \frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

If we take $\frac{\partial}{\partial x}$ of the first expression (from Faraday's Law) and use the equality of mixed partial derivatives, $\frac{\partial}{\partial t} \frac{\partial}{\partial x} = \frac{\partial}{\partial x} \frac{\partial}{\partial t}$, we get

$$\frac{\partial^2}{\partial x^2} E = -\frac{\partial}{\partial x} \frac{\partial}{\partial t} B \implies \frac{\partial^2}{\partial x^2} E = -\frac{\partial}{\partial t} \frac{\partial}{\partial x} B.$$

Using the second expression and canceling the sign gives a one dimensional wave equation.

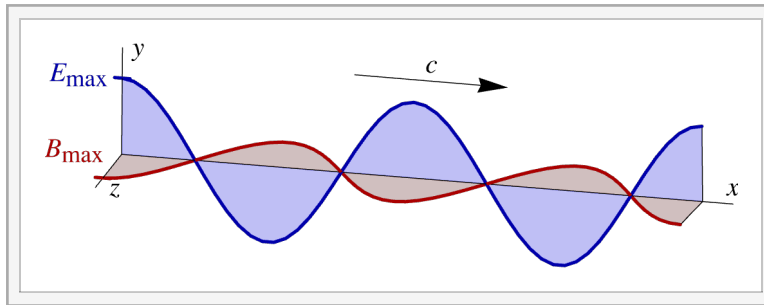
$$\frac{\partial^2}{\partial x^2} E = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E$$

It is clear that the above equation is of the form of the 1D wave equation $\frac{\partial^2}{\partial x^2} u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u$, if we associate the electric field E with the disturbance u and the wave speed v with

$$v = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

The constant c is the speed of light in a vacuum; it is a most fundamental constant in physics. When Maxwell first calculated this value for the speed of electromagnetic radiation, which was written in terms of the two electromagnetic constants ϵ_0 and μ_0 , and compared it with the previously measured value of the speed of light it was clear that visible light was an example of electromagnetic radiation. This was later verified as true.

Sinusoidal Plane Waves



Interactive Figure

If we choose a sinusoidal form of the electric field, which is in the y -direction, we get

$$E(x, t) = E_{\max} \cos(kx - \omega t).$$

Using one of our first order expressions $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ we can find the magnetic field by differentiating with respect to x and then integrating over t .

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \implies \frac{\partial B}{\partial t} = -\frac{\partial E}{\partial x} = k E_{\max} \sin(kx - \omega t) \implies B = \frac{k}{\omega} E_{\max} \cos(kx - \omega t).$$

It is clear the magnetic field, which recall is in the z -direction, is in phase with the electric field and its peak value is given by

$$B_{\max} = \frac{k}{\omega} E_{\max}.$$

Since the wave speed is c we can write

$$c = \frac{\omega}{k} = f \lambda$$

and this relates the electric and magnetic fields:

$$c = \frac{E_{\max}}{B_{\max}} = \frac{E}{B}.$$

Aside - Alternate Derivation of Wave Equation

Using the differential form of Maxwell's equations, discussed as an aside in Chapter G, we can derive a wave equation much more efficiently. Maxwell's equations in a vacuum ($\rho = 0$ and $\vec{J} = \vec{0}$) in differential form become:

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (\text{Gauss's Law})$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (\text{Gauss's Law for Magnetism})$$

$$\begin{aligned}\nabla \times \vec{B} &= \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E} \quad (\text{Ampere-Maxwell Law}) \\ -\nabla \times \vec{E} &= \frac{\partial}{\partial t} \vec{B} \quad (\text{Faraday's Law})\end{aligned}$$

We want to derive the three dimensional wave equation. Since the one-dimensional wave equation for $u(x, t)$ is $\frac{\partial^2}{\partial x^2} u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u$, it follows that its natural three-dimensional generalization for $u(x, y, z, t)$ is $\nabla^2 u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u$, where $\nabla^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) u$.

Begin by taking the curl of Faraday's Law.

$$-\nabla \times (\nabla \times \vec{E}) = \nabla \times \frac{\partial}{\partial t} \vec{B}$$

A vector calculus identity, that holds for any vector field F , is $\nabla^2 F = \nabla(\nabla \cdot F) - \nabla \times (\nabla \times F)$. Also, the equality of mixed partial derivatives $\left(\frac{\partial}{\partial x} \frac{\partial}{\partial t} = \frac{\partial}{\partial t} \frac{\partial}{\partial x} \right)$ implies that $\nabla \times \frac{\partial}{\partial t} \vec{B} = \frac{\partial}{\partial t} \nabla \times \vec{B}$. Applying these results gives

$$\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) = \frac{\partial}{\partial t} \nabla \times \vec{B}.$$

Using Gauss's Law and the Ampere-Maxwell Law we get an equation of the form of the three dimensional wave equation

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}.$$

From this we can read off the speed.

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

J.3 - Energy and Electromagnetic Radiation

Energy Density

The energy density u in an electromagnetic field can be written as a sum over electric and magnetic contributions

$$u = u_e + u_m = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2 \mu_0} B^2$$

We can find the average energy density u_{ave} using the fact that the fields are sinusoidal and that the average of \cos^2 is $\frac{1}{2}$.

$$u_{\text{ave}} = \frac{1}{2} \varepsilon_0 \frac{1}{2} E_{\text{max}}^2 + \frac{1}{2 \mu_0} \frac{1}{2} B_{\text{max}}^2$$

We can verify that the electric and magnetic contributions are equal, $u_{\text{ave},e} = u_{\text{ave},m}$.

$$\frac{E_{\text{max}}}{B_{\text{max}}} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \implies u_{\text{ave},e} = u_{\text{ave},m}$$

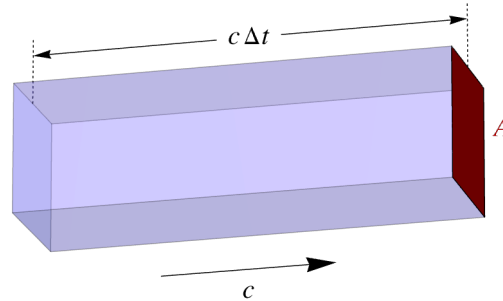
It follows that

$$u_{\text{ave}} = \frac{\varepsilon_0}{2} E_{\text{max}}^2 = \frac{B_{\text{max}}^2}{2 \mu_0} = \frac{E_{\text{max}} B_{\text{max}}}{2 \mu_0 c} = \frac{E_{\text{max}}^2}{2 \mu_0 c^2}$$

Intensity

Intensity I is a measure of the power (energy/time) per area. If A is the area of a surface normal to the radiation, Δt is a time, U is the energy passing through the surface and \mathcal{P} is the power ($U/\Delta t$) through the surface, then these are related by

$$I = \frac{\mathcal{P}}{A} = \frac{U}{A \Delta t}.$$



Consider all the radiation in the right cylinder (with any shape cross-section, even rectangular) and length $c\Delta t$. Since all the radiation, and thus all the energy, is flowing at speed c , it follows that all the energy in the cylinder passes A in Δt . The volume of the cylinder is $Ac\Delta t$, so

$$U = u_{\text{ave}} A c \Delta t \quad \text{and} \quad I = \frac{\mathcal{P}}{A} = \frac{U}{A \Delta t} = u_{\text{ave}} c$$

This gives several equivalent expressions for the intensity

$$I = u_{\text{ave}} c = \frac{E_{\text{max}}^2}{2 \mu_0 c} = \frac{E_{\text{max}} B_{\text{max}}}{2 \mu_0}$$

Poynting Vector

The Poynting vector \vec{S} is defined by

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}.$$

Application of the right hand rule shows that if the electric field is in the y -direction and the magnetic field is in the z -direction, then the Poynting vector is in the x -direction. Since the fields are perpendicular, the magnitude of the cross product is just the product of the fields. Taking the average of the magnitude we get

$$S_{\text{ave}} = \frac{\overline{EB}}{\mu_0} = \frac{E_{\text{max}} B_{\text{max}}}{2 \mu_0}.$$

This expression follows from averaging \cos^2 and getting $1/2$. It is clear that the average value of the Poynting vector is the intensity.

$$S_{\text{ave}} = I$$

The Poynting vector's direction is the direction of flow of electromagnetic waves and its magnitude is the intensity. This is quite generally the case, meaning that it is true independent of our plane wave assumptions. If $d\vec{A}$ is some infinitesimal surface and $d\mathcal{P}$ is the power (energy/time) passing through the surface then

$$d\mathcal{P} = \vec{S} \cdot d\vec{A}.$$

Example J.2 - An Isotropic Source

A light bulb radiates isotropically (the same in all directions). At a distance of 3.25 m, the intensity of the light is 0.064 W/m^2 .

(a) What is the total power radiated by the bulb?

Solution

The intensity is related to the power by $I = \mathcal{P}/A$. To find the correct expression for the area consider how the light is distributed. The light that leaves the bulb at one instant is, at a later instant, spread over the surface of a sphere; the surface area of a sphere

is then the relevant area. Here we know the intensity and the distance gives the area.

$$I = 0.064 \text{ W/m}^2, \quad r = 3.25 \text{ m} \implies A = 4\pi r^2 = 132.73 \text{ m}^2$$

We can now solve for the power.

$$I = \frac{\mathcal{P}}{A} \implies \mathcal{P} = IA = 8.49 \text{ W}$$

(b) What is the intensity of light at a distance of 6.5 m from the bulb?

Solution

\mathcal{P} is the total power radiated by the source, so that is the same regardless of the distance from the source, the bulb. We can then solve this using simple ratios.

$$I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\pi r^2} \implies \frac{I_2}{I_1} = \frac{1}{(r_2/r_1)^2} \implies I_2 = \frac{I_1}{(6.5 \text{ m}/3.25 \text{ m})^2} = \frac{0.064 \text{ W/m}^2}{4} = 0.016 \text{ W/m}^2$$

Example J.3 - A Laser Beam

A 125-mW He-Ne laser produces (polarized) 632.8-nm light with a beam of radius 0.58 mm.

(a) What is the frequency of light in the beam?

Solution

First we list everything we are given.

$$\mathcal{P} = 0.125 \text{ W}, \quad \lambda = 632.8 \times 10^{-9} \text{ m} \text{ and } r = 0.58 \times 10^{-3} \text{ m}$$

We also need the constants

$$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \text{ and } \mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$$

To find the frequency we only need the wavelength.

$$f = c/\lambda = 4.74 \times 10^{14} \text{ Hz}$$

(b) What is the intensity of light in the beam?

Solution

We find the intensity using $I = \mathcal{P}/A$, where the relevant area is the cross-sectional area of the beam. The light that leaves the laser at one instant is spread over the cross-section of the beam at a later instant.

$$A = \pi r^2 = 1.05683 \times 10^{-6} \text{ m}^2 \implies I = \mathcal{P}/A = 1.18 \times 10^5 \text{ W/m}^2$$

(c) What is the peak electric field inside the beam?

Solution

To find the peak electric field we use the intensity formula relating I to E_{max} .

$$I = \frac{E_{\text{max}}^2}{2\mu_0 c} \implies E_{\text{max}} = \sqrt{2\mu_0 c I} = 9443 \text{ V/m}$$

(d) What is the rms magnetic field inside the beam?

Solution

The ratio of the peak electric and magnetic fields is the speed of light. Since the fields are sinusoidal functions of time the rms field is related to the peak field as we saw last chapter.

$$c = \frac{E_{\text{max}}}{B_{\text{max}}} \implies B_{\text{max}} = \frac{E_{\text{max}}}{c} = 3.15 \times 10^{-5} \text{ T} \implies B_{\text{rms}} = \frac{B_{\text{max}}}{\sqrt{2}} = 2.23 \times 10^{-5} \text{ T}$$

J.4 - Momentum and Pressure

Momentum Carried by Radiation

Electromagnetic radiation carries energy. Since there is moving energy there is also momentum. To see this consider Einstein's famous formula $E = m c^2$. This was shown to be generally true in 1905, but in the case of electromagnetism an analogous expression had been derived previously. We are using U for energy, so let us write this as $U = m c^2$. Since momentum is $p = m v$ and the radiation is moving at c we can write $p = m c$. Combining these expressions we get

$$p = \frac{U}{c}.$$

This is the momentum carried by electromagnetic radiation.

Some comments should be made on the mass referred to above. The mass in $E = m c^2$ is known as the *relativistic mass*. This is to be distinguished from the *rest mass*. The tabulated values of the masses of particles are their rest masses. In relativity, a particle with a rest mass can never be accelerated to the speed of light, but it can reach a speed arbitrarily close to that of light. A particle of light is known as a photon; this is called a massless particle meaning that it has no rest mass. Massless particles must always move at c .

Momentum from Radiation Normally Incident on a Surface

If the radiation is normally incident on a surface we can derive simple expressions for the momentum given to the surface. First consider the case of a surface that is a perfect absorber. All of the momentum of the radiation is given to the surface, giving

$$p = \frac{U}{c} \quad (\text{perfect absorber}).$$

If the surface is a perfect reflector then the change in the momentum of the radiation is twice the value of the incident radiation. Recall that momentum is a vector and here we are subtracting two vectors in the opposite direction. Since momentum must be conserved, the change in the momentum of the radiation is equal (in magnitude) to the momentum given to the surface.

$$p = 2 \frac{U}{c} \quad (\text{perfect reflector})$$

We can interpolate between these two expressions. If κ is the fraction of energy reflected then the momentum gained by the surface is

$$p = (1 + \kappa) \frac{U}{c} \quad (\kappa \text{ is the fraction reflected}).$$

Pressure and Force on a Surface from Normally Incident Radiation

Newton's second law $F_{\text{net}} = \frac{d}{dt} \vec{p}$ relates force to momentum. The force can be related to the momentum in the case of normally incident radiation by the expression

$$F = \frac{p}{\Delta t}.$$

If A is the area of the surface and P is the pressure (using capital 'p' for pressure and lower case for momentum) then pressure is defined as force per area.

$$P = \frac{F}{A}.$$

We can then write the pressure P in terms of the momentum p .

$$P = \frac{p}{A \Delta t}$$

Using the definition of intensity

$$I = \frac{\mathcal{P}}{A} = \frac{U}{A \Delta t}.$$

We can turn the momentum expressions, which involve the energy U into expressions for the pressure involving the intensity I by dividing both sides of the momentum expressions by $A\Delta t$.

$$P = \frac{I}{c} \quad (\text{perfect absorber})$$

$$P = 2 \frac{I}{c} \quad (\text{perfect reflector})$$

$$P = (1 + \kappa) \frac{I}{c} \quad (\kappa \text{ is the fraction reflected})$$

Example J.4 - A Laser Beam (Continued)

A 125-mW He-Ne laser produces (polarized) 632.8-nm light with a beam of radius 0.58 mm.

(e) Suppose in space, this beam reflects normally off a metal surface that reflects 85% of the energy, what is the force of the beam on the surface?

Solution

The pressure is the force per area $P = F/A$ and the intensity is the power per area, $I = \mathcal{P}/A$. We can then relate the force to the power, directly. Note that this answer is independent of the cross-sectional area of the beam.

$$P = (1 + \kappa) \frac{I}{c} \implies F = P A = (1 + \kappa) \frac{\mathcal{P}/A}{c} A = (1 + \kappa) \frac{\mathcal{P}}{c}$$

We also know κ , the fraction of energy reflected.

$$\kappa = 85\% = 0.85 \implies F = (1 + \kappa) \frac{\mathcal{P}}{c} = 7.71 \times 10^{-10} \text{ N}$$

(f) How long would it take for the beam to give the surface in part (e) a momentum of $1.3 \text{ kg} \cdot \text{m/s}$?

Solution

The formula for the momentum to a surface gives us the total energy.

$$p = 1.3 \text{ kg} \cdot \text{m/s} \quad \text{and} \quad p = (1 + \kappa) \frac{U}{c} \implies U = \frac{p c}{1 + \kappa} = 2.1081 \times 10^8 \text{ J}$$

The power is the energy per time and we can solve for the time.

$$\mathcal{P} = \frac{U}{t} \implies t = \frac{U}{\mathcal{P}} = 1.69 \times 10^9 \text{ s} = 53.4 \text{ yr}$$

Clearly, pushing something with a laser would take some patience!

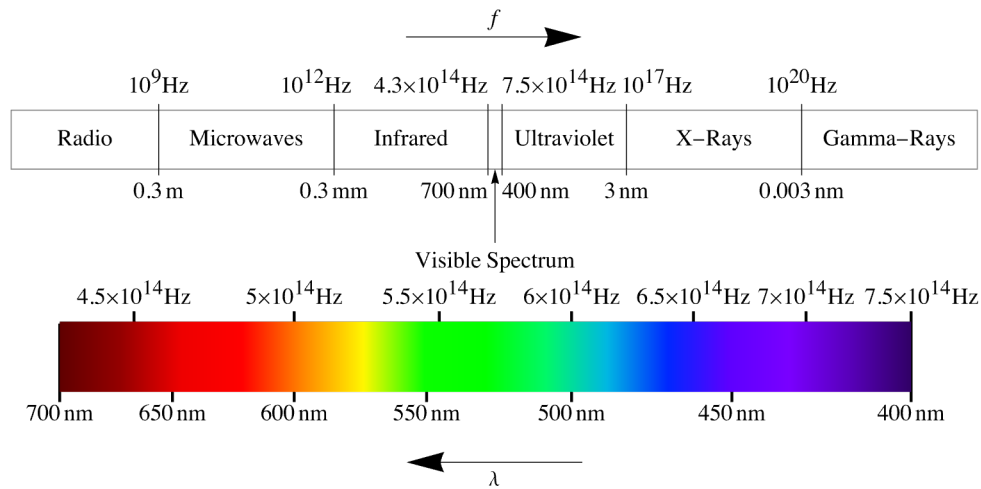
J.5 - The Electromagnetic Spectrum

The Full Spectrum

Electromagnetic radiation can be described by its wavelength and frequency. Since frequency and wavelength are related by

$$f \lambda = c$$

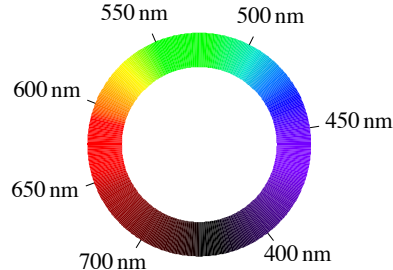
for radiation in a vacuum, it follows that all electromagnetic radiation can be written along a line of increasing frequency and decreasing wavelength.



Long wavelength waves are radio waves. Wavelengths shorter than around 0.3 meter are usually labeled microwaves. Smaller than about 1 mm start the infrared (IR) waves. At 700 nm we get into the visible spectrum which is a narrow band of wavelengths down to 400 nm. Below 400 nm to around 3 nm are the ultraviolet (UV) waves. Wavelengths smaller than that are called X-rays and the small wavelength limit, beyond around 0.003 nm, are called gamma rays.

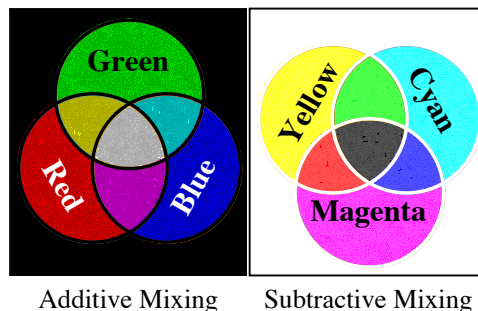
The Visible Spectrum and Primary Colors

The visible spectrum consists of the narrow band of wavelengths between 400 and 700 nm. In order of decreasing wavelength (increasing frequency) we have ROYGBIV: Red, Orange, Yellow, Green, Blue, Indigo and Violet. There is approximately a factor of two of wavelengths (and frequencies) we can see; this is crucial to our perception of light. With sound we can hear many octaves, where an octave higher corresponds to doubling a frequency. As you go through the “do-re-mi” scale from the low “do” to the high “do”, you jump up an octave, doubling the frequency. With light we can see the equivalent of only one octave of light. Our brain matches the ends of the visible spectrum into a circle; violet appears as a reddish blue. If we perceived sound similarly, the low “do” and high “do” would sound the same.



Notice that the visible spectrum is not quite a factor of two of wavelengths and frequencies. There is a missing color, magenta. Although we may perceive magenta as a combinations of other colors, there is no pure color, meaning color of a single wavelength, corresponding to magenta.

Matching the ends of the visible spectrum together makes it possible to represent colors as combinations of three primary colors. There are two notions of primary colors: additive mixing and subtractive mixing. Additive mixing is used with computer monitors and television screens. We begin with black and add colors. The additive primary colors are Red, Green and Blue. Combining all three we can get white as shown. Subtractive mixing is where we begin with white and remove colors. This is used when mixing paints or for color printers. Here the primary colors are Cyan (a blue-green color), Magenta (a reddish violet) and Yellow. Removing all three subtractive colors gives black.



Example 10.5 - Frequencies and Wavelengths

(a) Modern microwave ovens operate at a frequency of 2450 MHz. What is the wavelength of the radiation?

Solution

$$c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}} \text{ and } f = 2450 \text{ MHz} = 2450 \times 10^6 \text{ Hz} \implies \lambda = \frac{c}{f} = 0.122 \text{ m} = 12.2 \text{ cm}$$

There is a mesh on the door of all microwaves to keep the microwaves inside. But we can see inside, so clearly some electromagnetic radiation passes through. Why can light pass through and microwaves not? This has to do with wavelengths. The wavelength of the microwaves is much larger than the spacing on the mesh, so the waves are reflected back as if the mesh were a uniform conducting sheet. The spacing on the mesh is much larger than the wavelength of visible light so that passes through easily.

(b) Radio stations identify by their frequency, FM stations give their frequency in MHz, ranging from 87.5 to 108 MHz. AM stations are in kHz, between 525 and 1705 kHz. What are the wavelengths of an FM station at 90.9 MHz and an AM station at 1705 kHz.

Solution

$$f = 90.9 \times 10^6 \text{ Hz} \implies \lambda = \frac{c}{f} = 3.30 \text{ m} \text{ and } f = 1705 \times 10^3 \text{ Hz} \implies \lambda = \frac{c}{f} = 176 \text{ m}$$

(c) What is the frequency of an X-ray with a wavelength of 0.020 nm?

Solution

$$\lambda = 0.020 \times 10^{-9} \text{ m} \implies f = \frac{c}{\lambda} = 1.5 \times 10^{19} \text{ Hz}$$